Lie Groups and Algebraic Groups

21–22 July 2011

Department of Mathematics
University of Bielefeld
Lecture Room V3-201

This workshop is part of the conference program of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the University of Bielefeld

Organizers: Herbert Abels and Ernest Vinberg

http://www.math.uni-bielefeld.de/sfb701/2011_LieGroupsandAlgebraicGroups
Program

All the talks take place in the "Common Room" (V3-201).

Thursday, July 21st, 2011

9:30 – 10:30  **W. de Graaf** (Trento)
An algorithm to compute the closure of a nilpotent orbit of a theta-group

10:40 – 11:40  **S. Cupit-Foutou** (Cologne)
Wonderful varieties

11:40 – 12:00  *Coffee break*

12:00 – 13:00  **R. Avdeev** (Moscow)
Harmonic analysis on spherical homogeneous spaces

13:00 – 15:00  *Lunch break*

15:00 – 15:30  *Coffee break*

15:30 – 16:30  **P. Littelmann** (Cologne)
PBW degeneration: representations, flag varieties, polytopes and combinatorics in type A and C

16:40 – 17:40  **A. Felikson, P. Tumarkin** (Bremen)
Cluster algebras and triangulated orbifolds

Friday, July 22nd, 2011

9:30 – 10:30  **S. Gindikin** (Rutgers University)
Geometry of horospheres and the horospherical transform

10:40 – 11:40  **J. Hilgert** (Paderborn)
Patterson-Sullivan distributions in higher rank

11:40 – 12:00  *Coffee break*

12:00 – 13:00  **O. Yakimova** (Erlangen)
One-parameter contractions of Lie-Poisson brackets

13:00 – 15:00  *Lunch break*

15:00 – 15:30  *Coffee break*

15:30 – 16:30  **E. Vishnyakova** (Bonn)
Non-split homogeneous supermanifolds

16:40 – 17:40  **V. S. Zhgoon** (Moscow) (based on joint work with D.A. Timashev)
On complexity of Lagrangian subvarieties in Hamiltonian varieties
Abstracts

Roman Avdeev (Moscow State University)

Harmonic analysis on spherical homogeneous spaces

Let $G$ be a semisimple complex algebraic group and $H$ a closed subgroup of it. An important problem in the theory of algebraic transformation groups is to compute the spectra of the natural representations of $G$ on spaces of regular sections of homogeneous line bundles over the homogeneous space $G/H$. According to a result of Kimelfeld and Vinberg, spherical subgroups $H$ are characterized by the property that all the above-mentioned representations are multiplicity-free. In this case, the whole collection of their spectra is uniquely determined by the so-called extended weight semigroup of $G/H$. In the talk we shall discuss some properties of this semigroup and some approaches to computation of it.

Stéphanie Cupit-Foutou (University of Cologne)

Wonderful varieties

Wonderful varieties form an important class of compactifications of homogeneous spaces. They include in particular flag varieties and DeConcini-Procesi symmetric varieties. My talk deals with the problem of classifying wonderful varieties by means of combinatorial objects, as proposed by Luna.

Anna Felikson, Pavel Tumarkin (Jacobs University of Bremen)

Cluster algebras and triangulated orbifolds

Cluster algebras were introduced by Fomin and Zelevinsky in 2000, and since then appear in various contexts. It was shown by Fomin, Shapiro and Thurston that a large class of cluster algebras can be constructed via triangulated bordered surfaces with marked points. After reviewing their construction, we extend it to triangulated orbifolds, and show some applications, such as computation of growth of cluster algebras. The work is joint with Michael Shapiro.

Simon Gindikin (Rutgers University)

Geometry of horospheres and the horospherical transform.

The Plancherel formula for some symmetric spaces is equivalent to the inversion of the horospherical transform. We will show that this inversion problem is equivalent to its flat analogue.

Willem de Graaf (University of Trento)

An algorithm to compute the closure of a nilpotent orbit of a theta-group

Theta-groups are reductive algebraic groups that arise from gradings of semisimple Lie algebras. They were introduced and studied by Vinberg in the 70’s. They have many interesting properties. One of them is that they have a finite number of nilpotent orbits. The closure of a nilpotent orbit consists of a number of nilpotent orbits. In this talk we will outline an algorithm for deciding whether one nilpotent orbit lies in the closure of another. Also some results obtained with the implementation of the algorithm will be discussed. This is joint work with E. Vinberg and O. Yakimova.
Joachim Hilgert (University of Paderborn)

Patterson-Sullivan distributions in higher rank

For a compact locally symmetric space of non-positive curvature, we consider sequences of normalized joint eigenfunctions which belong to the principal spectrum of the algebra of invariant differential operators. Using an h-pseudodifferential calculus, we define and study lifted quantum limits as weak-limit points of Wigner distributions. The Helgason boundary values of the eigenfunctions allow us to construct Patterson–Sullivan distributions on the space of Weyl chambers. These distributions are asymptotic to lifted quantum limits and satisfy additional invariance properties, which makes them useful in the context of quantum ergodicity. Our results generalize results for compact hyperbolic surfaces obtained by Anantharaman and Zelditch.

Peter Littelmann (University of Cologne)

PBW degeneration: representations, flag varieties, polytopes and combinatorics in type A and C

This is a report on joint work of Michael Finkelberg, Evgeny Feigin, Ghislain Fourier, Peter Littelmann. The PBW filtration on a highest weight representation of a simple Lie algebra is induced by the standard (degree) filtration on the universal enveloping algebra of lowering operators. The associated graded space carries a structure of a representation of the degenerate Lie algebra and the degenerate Lie group. We will describe these representations for the Lie algebras of type A and C. We will also define the degenerate analogues of the flag varieties. We will give an explicit description of these singular varieties, construct desingularizations and derive a formula for the q-characters of the highest weight g-modules.

Elizaveta Vishnyakova (Ruhr-Universität Bochum)

Non-split homogeneous supermanifolds

It is well known that any smooth supermanifold is split by the Batchelor Theorem. This assertion is false in the complex case. For example almost all classical flag supermanifolds are non-split. Our talk will be devoted to the question: How to find out, whether a complex homogeneous supermanifold is split or non-split?

Oksana Yakimova (FAU Erlangen-Nuernberg)

One-parameter contractions of Lie-Poisson brackets. (Partly based on a joint paper with D. Panyushev)

Contractions provide a way to replace a simple Lie algebra \( g \) by a semidirect product of its subalgebra and a complementary subspace, which becomes an Abelian ideal. In some cases the Poisson tensor of \( g \) behaves well under a contraction and this allows us to get a description of the symmetric invariants for the resulting Lie algebra. Two contractions will be considered, one is related to a symmetric decomposition \( g_0 + g_1 \) of \( g \) and it was studied before by D. Panyushev. The second one was recently introduced by E. Feigin. Here a subalgebra is a Borel and the complementary subspace is the nilpotent radical of an opposite Borel.

Vladimir Zhgoon (Institute of System Studies, Moscow)

On complexity of Lagrangian subvarieties in Hamiltonian varieties.

(based on joint work with D.A. Timashev) Let \( G \) be a reductive group over an algebraically closed field of characteristic zero, and let \( X \) be a symplectic \( G \)-variety equipped with a moment map. We prove that all \( G \)-invariant Lagrangian subvarieties of \( X \) have the same complexity and rank. We also give a calculation of the closure of the image of the moment map that generalizes well-known results on the cotangent bundles of \( G \)-varieties. We note that this is a generalization of a result of D.I.Panyushev, who proved that for a \( G \)-invariant subvariety \( Y \) of a \( G \)-variety \( X \) the conormal bundle of \( Y \) in \( X \) has the same complexity as \( X \).
Registered participants

Herbert Abels (University of Bielefeld)
Roman Avdeev (Moscow State University)
Stéphanie Cupit-Foutou (University of Cologne)
Alexander Elashvili (A.M. Razmadze Institute of Mathematik, Tbilisi)
Anna Felikson (Jacobs University of Bremen)
Simon Gindikin (Rutgers University)
Willem de Graaf (University of Trento)
Joachim Hilgert (University of Paderborn)
Michael Hinz (Universität Jena)
Elena Klimenko (University of Duesseldorf)
Peter Littelmann (University of Cologne)
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(as of 15th July 2011)