

# TAMENESS AND COMPLEXITY OF FINITE GROUP SCHEMES

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ABSTRACT. Using a representation-theoretic interpretation of support varieties due to Friedlander-Pevtsova [16], we show that the complexity of tame blocks of finite group schemes is bounded by 2. In this context, our result salvages a theorem by Rickard [21], whose proof is flawed.

## 1. INTRODUCTION AND PRELIMINARIES

In his paper [21] J. Rickard established an important result implying that the complexity  $\text{cx}_\Lambda(M)$  of a finite-dimensional module  $M$  over a self-injective tame algebra  $\Lambda$  is bounded by 2. His geometric arguments require a test for wildness [21, Lemma 1], which was recently falsified by K. Erdmann, A. Skowroński and G. Zvara. As many classification results (cf. [11, 12, 13, 14, 18]) concerning reduced enveloping algebras and infinitesimal group schemes of tame representation type ultimately rest on the boundedness of complexity, it seems desirable to salvage Rickard's Theorem in these contexts. The main result of this article, Theorem 3.1, accomplishes this for blocks of cocommutative Hopf algebras, or algebras of measures associated to finite group schemes. Our methods are based on J. Carlson's seminal papers [4, 5] concerning support varieties for finite groups as well as recent work by Friedlander and Pevtsova [16].

Throughout, we will be working over an algebraically closed field  $k$  of characteristic  $p > 0$ . If  $f_1, \dots, f_n \in k[V]$  are elements of the coordinate ring of an affine variety  $V$ , then

$$Z(f_1, \dots, f_n) := \{v \in V ; f_i(v) = 0, 1 \leq i \leq n\}$$

denotes the closed subvariety of their common zeros. Let  $\mathcal{V}$  be a conical affine variety, that is, an affine variety, whose coordinate ring  $k[\mathcal{V}]$  affords a gradation

$$k[\mathcal{V}] = \bigoplus_{i \geq 0} k[\mathcal{V}]_i$$

with  $k[\mathcal{V}]_0 = k$ . A closed subset  $V \subset \mathcal{V}$  is called *conical* if there exist homogeneous elements  $f_1, \dots, f_n \in k[\mathcal{V}]$  such that

$$V = Z(f_1, \dots, f_n).$$

In that case, the comorphism  $\iota^* : k[\mathcal{V}] \rightarrow k[V]$  is homogeneous of degree 0.

We begin with the following elementary Lemma.

**Lemma 1.1.** *Let  $V \subset \mathcal{V}$  be a conical variety of dimension  $n \geq 2$ . Then there exist homogeneous elements  $(f_\alpha)_{\alpha \in k} \in k[\mathcal{V}]$  of degree  $d > 0$ , such that*

- (a)  $\dim Z(f_\alpha) \cap V = n - 1$  for  $\alpha \in k$ , and
- (b)  $\dim Z(f_\alpha) \cap Z(f_\beta) \cap V = n - 2$  for  $\alpha \neq \beta \in k$ .

*Proof.* The Noether Normalization Lemma [8, (13.3)] provides homogeneous elements  $f_1, \dots, f_n \in k[\mathcal{V}]$  of degree  $d > 0$  such that the images  $\bar{f}_i \in k[V]$  have the following properties:

- (1) The set  $\{\bar{f}_1, \dots, \bar{f}_n\}$  is algebraically independent, and
- (2)  $k[V]$  is a finitely generated  $k[\bar{f}_1, \dots, \bar{f}_n]$ -module.

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As a result, the morphism

$$\varphi : V \longrightarrow \mathbb{A}^n \ ; \ v \mapsto (f_1(v), \dots, f_n(v))$$

is dominant and finite. In particular, the map  $\varphi$  is closed and surjective, and we have

$$(*) \quad \dim \varphi^{-1}(W) = \dim W$$

for any closed subset  $W \subset \mathbb{A}^n$  (cf. [8, Cor.9.3]).

Now let  $\text{pr} : \mathbb{A}^n \longrightarrow \mathbb{A}^2$  be the projection onto the first two coordinates, and consider  $\Psi := \text{pr} \circ \varphi$ . Thus,

$$\Psi : V \longrightarrow \mathbb{A}^2 \ ; \ v \mapsto (f_1(v), f_2(v))$$

is a surjective morphism.

Given  $\alpha \in k$ , we put  $\mathcal{L}_\alpha := \{(x, \alpha x) \ ; \ x \in k\}$ . Then  $Q_\alpha := \text{pr}^{-1}(\mathcal{L}_\alpha) \subset \mathbb{A}^n$  is a subspace of dimension  $n - 1$ , and  $(*)$  yields

$$\dim Z(f_2 - \alpha f_1) \cap V = \dim \Psi^{-1}(\mathcal{L}_\alpha) = \dim \varphi^{-1}(Q_\alpha) = n - 1.$$

In particular,  $f_\alpha := f_2 - \alpha f_1 \in k[\mathcal{V}]_d$  is a non-zero homogeneous element of degree  $d$ . For  $\alpha \neq \beta \in k$  we have

$$Z(f_\alpha) \cap Z(f_\beta) \cap V = \Psi^{-1}(\mathcal{L}_\alpha) \cap \Psi^{-1}(\mathcal{L}_\beta) = \Psi^{-1}(\mathcal{L}_\alpha \cap \mathcal{L}_\beta) = \Psi^{-1}(0).$$

Since  $\ker \text{pr} \subset \mathbb{A}^n$  is a subspace of dimension  $n - 2$ , another application of  $(*)$  implies  $\dim \Psi^{-1}(0) = n - 2$ , as desired.  $\square$

## 2. REALIZABILITY OF CONICAL VARIETIES

Let  $\mathcal{G}$  be a finite  $k$ -group scheme. By general theory (cf. [20, (I.8.6)]), the module category of  $\mathcal{G}$  is equivalent to that of the algebra  $M(\mathcal{G})$  of measures on  $\mathcal{G}$ . By definition,  $M(\mathcal{G})$  is the dual of the coordinate ring of  $\mathcal{G}$ . It coincides with the group algebra  $k[\mathcal{G}(k)]$  of the group of  $k$ -rational points of  $\mathcal{G}$  in case  $\mathcal{G}$  is reduced, and with the algebra  $\text{Dist}(\mathcal{G})$  of distributions in case  $\mathcal{G}$  is infinitesimal. Given a finite group  $G$ , we denote by  $G_k$  the reduced finite group scheme satisfying  $G_k(k) = G$ .

In this Section we define for every block  $\mathcal{B} \subset M(\mathcal{G})$  a conical variety  $\mathcal{V}_\mathcal{B}$ , such that the closed, conical subvarieties of  $\mathcal{V}_\mathcal{B}$  occur as support varieties of  $\mathcal{B}$ -modules. Given a finite dimensional  $\mathcal{G}$ -module  $M$ , the Yoneda composition induces a canonical homomorphism

$$\Phi_M : H^{\text{ev}}(\mathcal{G}, k) \longrightarrow \text{Ext}_{\mathcal{G}}^{\text{ev}}(M, M) \ ; \ [f] \mapsto [f \otimes \text{id}_M]$$

of  $k$ -algebras. Thanks to the Friedlander-Suslin Theorem [17, Theorem 1.1] the algebra  $H^{\text{ev}}(\mathcal{G}, k)$  is finitely generated, and  $\text{Ext}_{\mathcal{G}}^{\bullet}(M, M)$  is a finite  $H^{\text{ev}}(\mathcal{G}, k)$ -module. The *cohomological support variety*  $\mathcal{V}_\mathcal{G}(M)$  is the variety associated to the ideal  $\sqrt{\ker \Phi_M}$ . Since  $\Phi_M$  is a graded homomorphism, the variety  $\mathcal{V}_\mathcal{G}(M) \subset \mathcal{V}_\mathcal{G}(k)$  is conical. We put  $\mathcal{V} := \mathcal{V}_\mathcal{G}(k)$ , so that the affine  $k$ -algebra  $k[\mathcal{V}] := H^{\text{ev}}(\mathcal{G}, k)/\sqrt{(0)}$  is graded via  $k[\mathcal{V}] = \bigoplus_{i \geq 0} k[\mathcal{V}]_{2i}$ . Given  $\zeta_1, \dots, \zeta_n \in H^{\text{ev}}(\mathcal{G}, k)$ , we denote by  $Z(\zeta_1, \dots, \zeta_n)$  the set of common zeros of the corresponding images  $\bar{\zeta}_1, \dots, \bar{\zeta}_n \in k[\mathcal{V}]$ .

Let  $\mathcal{B} \subset M(\mathcal{G})$  be a block,  $\{S_1, \dots, S_n\}$  a complete set of representatives of the simple  $\mathcal{B}$ -modules. Then

$$\mathcal{V}_\mathcal{B} := \bigcup_{i=1}^n \mathcal{V}_\mathcal{G}(S_i)$$

is the *support variety* of the block  $\mathcal{B}$ . By general properties of support varieties [3, §5.7], we have

$$\mathcal{V}_\mathcal{G}(M) \subset \mathcal{V}_\mathcal{B}$$

for every finite-dimensional  $\mathcal{B}$ -module  $M$ .

In the sequel,  $\Omega_\mathcal{G}$  denotes the Heller operator of the category of finite dimensional  $\mathcal{G}$ -modules (see [2, p.29]). According to [2, (2.5.4)] an element  $\zeta \in H^{2n}(\mathcal{G}, k)$  corresponds to a linear map

$\hat{\zeta} : \Omega_{\mathcal{G}}^{2n}(k) \longrightarrow k$ , whose kernel we denote by  $L_{\zeta}$ . We require the following generalization of [5, (2.3)]:

**Lemma 2.1.** *We have  $\mathcal{V}_{\mathcal{G}}(L_{\zeta}) = Z(\zeta)$  for any  $\zeta \in H^{2n}(\mathcal{G}, k)$ .*

*Proof.* Let  $\alpha : M((\mathbb{Z}/(p))_k) \longrightarrow M(\mathcal{G})$  be an abelian  $p$ -point of  $\mathcal{G}$  (cf. [16, §3]), and consider the induced map  $\alpha^* : H^{\text{ev}}(\mathcal{G}, k) \longrightarrow H^{\text{ev}}((\mathbb{Z}/(p))_k, k)$ . As argued in the proof of [16, (2.3)], we have  $\zeta \in \ker \alpha^*$  if and only if the pull-back  $L_{\zeta}|_{M((\mathbb{Z}/(p))_k)}$  of  $L_{\zeta}$  along  $\alpha$  is not projective. Thanks to [16, (5.1)], the latter condition is equivalent to  $\ker \alpha^* \supset \ker \Phi_{L_{\zeta}}$ , so that  $(\zeta) \subset \ker \alpha^* \Leftrightarrow \ker \Phi_M \subset \ker \alpha^*$ . In view of [16, (4.11)], this implies  $\text{Proj}(Z(\zeta)) = \text{Proj}(\mathcal{V}_{\mathcal{G}}(L_{\zeta}))$ , whence  $Z(\zeta) = \mathcal{V}_{\mathcal{G}}(L_{\zeta})$ .  $\square$

The following results, which, by virtue of [15] also obtain in the context reduced enveloping algebras, refine [22, (7.5)].

**Proposition 2.2.** *Let  $\mathcal{B} \subset M(\mathcal{G})$  be a block,  $M$  a  $\mathcal{B}$ -module. Given  $\zeta \in H^{2n}(\mathcal{G}, k)$ , there exists a  $\mathcal{B}$ -module  $N$  of dimension  $\dim_k N \leq (\dim_k M)(\dim_k \Omega_{\mathcal{G}}^{2n}(k))$  such that  $\mathcal{V}_{\mathcal{G}}(N) = Z(\zeta) \cap \mathcal{V}_{\mathcal{G}}(M)$ .*

*Proof.* Upon tensoring the exact sequence

$$(0) \longrightarrow L_{\zeta} \longrightarrow \Omega_{\mathcal{G}}^{2n}(k) \xrightarrow{\hat{\zeta}} k \longrightarrow (0)$$

with the  $\mathcal{G}$ -module  $M$ , we obtain an exact sequence

$$(*) \quad (0) \longrightarrow L_{\zeta} \otimes_k M \longrightarrow \Omega_{\mathcal{G}}^{2n}(k) \otimes_k M \longrightarrow M \longrightarrow (0).$$

By general theory [2, (3.1.6)], there exists a projective  $\mathcal{G}$ -module  $P$  such that  $\Omega_{\mathcal{G}}^{2n}(M) \oplus P \cong \Omega_{\mathcal{G}}^{2n}(k) \otimes_k M$ . Let  $e \in M(\mathcal{G})$  be the central primitive idempotent associated to  $\mathcal{B}$ . Since  $e \cdot M = M$  and  $e \cdot \Omega_{\mathcal{G}}^{2n}(M) = \Omega_{\mathcal{G}}^{2n}(M)$ , multiplication of  $(*)$  by  $e$  and  $1 - e$  yields short exact sequences

$$(0) \longrightarrow e \cdot (L_{\zeta} \otimes_k M) \longrightarrow \Omega_{\mathcal{G}}^{2n}(M) \oplus e \cdot P \longrightarrow M \longrightarrow (0),$$

and

$$(0) \longrightarrow (1 - e) \cdot (L_{\zeta} \otimes_k M) \longrightarrow (1 - e) \cdot P \longrightarrow (0) \longrightarrow (0).$$

As  $(1 - e) \cdot P$  is a projective  $\mathcal{G}$ -module, we conclude that  $(1 - e) \cdot (L_{\zeta} \otimes_k M)$  is projective. Consequently,  $N := e \cdot (L_{\zeta} \otimes_k M)$  is a  $\mathcal{B}$ -module of dimension  $\dim_k N \leq (\dim_k M)(\dim_k \Omega_{\mathcal{G}}^{2n}(k))$  such that  $L_{\zeta} \otimes_k M = N \oplus P'$  for some projective  $\mathcal{G}$ -module  $P'$ . Thanks to Lemma 2.1, we have  $\mathcal{V}_{\mathcal{G}}(L_{\zeta}) = Z(\zeta)$ . We may now apply [16, (5.6(3))] and [16, (4.11)] to obtain

$$Z(\zeta) \cap \mathcal{V}_{\mathcal{G}}(M) = \mathcal{V}_{\mathcal{G}}(L_{\zeta} \otimes_k M) = \mathcal{V}_{\mathcal{G}}(N),$$

as desired.  $\square$

**Corollary 2.3.** *Let  $\mathcal{B} \subset M(\mathcal{G})$  be a block,  $V \subset \mathcal{V}_{\mathcal{B}}$  be a closed conical subvariety of  $\mathcal{V}$ . Then there exists a  $\mathcal{B}$ -module  $M$  such that*

$$V = \mathcal{V}_{\mathcal{G}}(M).$$

*Proof.* By assumption there exist homogeneous elements  $\zeta_1, \dots, \zeta_{\ell} \in H^{\text{ev}}(\mathcal{G}, k)$  such that  $V = \bigcap_{j=1}^{\ell} Z(\zeta_j) \cap \mathcal{V}_{\mathcal{B}}$ . General properties of support varieties [3, §5.7] yield  $\mathcal{V}_{\mathcal{B}} = \mathcal{V}_{\mathcal{G}}(M_0)$ , where  $M_0 := \bigoplus_{i=1}^n S_i$  is a  $\mathcal{B}$ -module. Proposition 2.2 now provides a finite sequence  $(M_j)_{1 \leq j \leq \ell}$  of  $\mathcal{B}$ -modules with  $\mathcal{V}_{\mathcal{G}}(M_j) = Z(\zeta_j) \cap \mathcal{V}_{\mathcal{G}}(M_{j-1})$ . In particular, we obtain  $\mathcal{V}_{\mathcal{G}}(M_{\ell}) = \bigcap_{j=1}^{\ell} Z(\zeta_j) \cap \mathcal{V}_{\mathcal{B}} = V$ .  $\square$

## 3. TAME BLOCKS OF FINITE GROUP SCHEMES

In this Section we give a proof of Rickard's Theorem [21, Theorem 2] for blocks of finite group schemes. As noted before, the arguments may be transferred *mutatis mutandis* to the context reduced enveloping algebras.

Given a finite dimensional  $k$ -algebra  $\Lambda$  and a natural number  $d > 0$ , we denote by  $\text{mod}^d \Lambda$  the affine variety of  $d$ -dimensional  $\Lambda$ -modules. The group  $\text{GL}_d(k)$  acts on  $\text{mod}^d \Lambda$  such that the orbits are the isomorphism classes of  $d$ -dimensional  $\Lambda$ -modules. The notion of *representation type* is related to parametrizations of the constructible subset  $\text{ind}_\Lambda^d \subset \text{mod}_\Lambda^d$  of indecomposable  $\Lambda$ -modules of dimension  $d$ : For every  $d > 0$ , let  $C_d \subset \text{mod}_\Lambda^d$  be a closed subset of minimal dimension subject to  $\text{ind}_\Lambda^d \subset \text{GL}_d(k) \cdot C_d$ . The algebra  $\Lambda$  is *tame*, if  $\dim C_d \leq 1$  for all  $d > 0$ . Otherwise  $\Lambda$  is referred to as *wild*. In view of Drozd's work [6, 7] the module category of a wild algebra is at least as complicated as that of any other algebra. Thus, there is no hope of classifying the indecomposables of such algebras.

In the representation theory of infinitesimal group schemes, support varieties and rank varieties play an important rôle in the determination of the components of the *stable Auslander-Reiten quiver* of  $\text{Dist}(\mathcal{G})$  (cf. [10]). Using abelian  $p$ -points, the results of [10, §1-§3] generalize to finite group schemes. We shall only require one fact concerning a certain type of AR-components, the so-called *homogeneous tubes*, and refer the interested reader to [1, 9] for more details.

Given a finite-dimensional  $\mathcal{G}$ -module  $M$ , we denote its *complexity* by  $\text{cx}_{\mathcal{G}}(M)$  (cf. [3, p.157f]). By general theory, we have  $\text{cx}_{\mathcal{G}}(M) = \dim \mathcal{V}_{\mathcal{G}}(M)$  for every  $\mathcal{G}$ -module  $M$  (cf. for instance [3, (5.7.2)]). All modules belonging to an AR-component  $\Theta$  have the same complexity. If  $\Theta$  is a homogeneous tube, then the Auslander-Reiten translate coincides on  $\Theta$  with the identity, and every module  $M$  belonging to  $\Theta$  satisfies

$$\Omega_{\mathcal{G}}^2(M) \cong M \otimes_k k_{\zeta}.$$

Here,  $\zeta : M(\mathcal{G}) \rightarrow k$  is the *modular function* of the Hopf algebra  $M(\mathcal{G})$  (see [20, (I.8.8)]). Consequently, the non-projective module  $M$  has a minimal projective resolution, whose constituents have uniformly bounded dimensions. Accordingly, we have

$$\text{cx}_{\mathcal{G}}(M) = 1 \quad \forall M \in \Theta.$$

We are now in a position to establish the following criterion for wildness:

**Theorem 3.1.** *Let  $\mathcal{B} \subset M(\mathcal{G})$  be a block. If  $\dim \mathcal{V}_{\mathcal{B}} \geq 3$ , then  $\mathcal{B}$  is wild.*

*Proof.* As before, we let  $\{S_1, \dots, S_n\}$  be a complete set of representatives for the isoclasses of simple  $\mathcal{B}$ -modules. Setting  $M_0 := \bigoplus_{i=1}^n S_i$ , we recall that  $\mathcal{V}_{\mathcal{B}} = \mathcal{V}_{\mathcal{G}}(M_0)$ . Since  $\mathcal{V}_{\mathcal{B}}$  is a conical subvariety of  $\mathcal{V}$ , Lemma 1.1 provides homogeneous elements  $(\zeta_{\alpha})_{\alpha \in k}$  of  $H^{\text{ev}}(\mathcal{G}, k)$  of degree  $2d > 0$  such that

- (a)  $\dim Z(\zeta_{\alpha}) \cap \mathcal{V}_{\mathcal{B}} = \dim \mathcal{V}_{\mathcal{B}} - 1 \geq 2$  for  $\alpha \in k$ , and
- (b)  $\dim Z(\zeta_{\alpha}) \cap Z(\zeta_{\beta}) \cap \mathcal{V}_{\mathcal{B}} = \dim \mathcal{V}_{\mathcal{B}} - 2$  for  $\alpha \neq \beta \in k$ .

Proposition 2.2 furnishes  $\mathcal{B}$ -modules  $N_{\alpha}$  of dimension  $\dim_k N_{\alpha} \leq (\dim_k \Omega_{\mathcal{G}}^{2d}(k))(\dim_k M_0) =: q_0$  such that

$$\mathcal{V}_{\mathcal{G}}(N_{\alpha}) = Z(\zeta_{\alpha}) \cap \mathcal{V}_{\mathcal{B}} \quad \forall \alpha \in k.$$

If  $N_{\alpha} = \bigoplus_{j=1}^{m_{\alpha}} N_{\alpha,j}$  is a decomposition into indecomposable  $\mathcal{B}$ -modules, then we have  $\mathcal{V}_{\mathcal{G}}(N_{\alpha}) = \bigcup_{j=1}^{m_{\alpha}} \mathcal{V}_{\mathcal{G}}(N_{\alpha,j})$ , and there thus exists an indecomposable summand  $X_{\alpha} := N_{\alpha,j(\alpha)}$  with

$$\mathcal{V}_{\mathcal{G}}(X_{\alpha}) \subset Z(\zeta_{\alpha}) \cap \mathcal{V}_{\mathcal{B}} \quad \text{and} \quad \dim \mathcal{V}_{\mathcal{G}}(X_{\alpha}) = \dim \mathcal{V}_{\mathcal{B}} - 1.$$

If  $\alpha \neq \beta$ , then the assumption  $\mathcal{V}_{\mathcal{G}}(X_{\alpha}) = \mathcal{V}_{\mathcal{G}}(X_{\beta})$  implies

$$\mathcal{V}_{\mathcal{G}}(X_{\alpha}) \subset Z(\zeta_{\alpha}) \cap Z(\zeta_{\beta}) \cap \mathcal{V}_{\mathcal{B}},$$

which contradicts property (b). As a result, the varieties  $\mathcal{V}_{\mathcal{G}}(X_{\alpha})$  are pairwise distinct, so that the indecomposable  $\mathcal{B}$ -modules  $X_{\alpha}$  are pairwise non-isomorphic. Since  $\dim_k X_{\alpha} \leq q_0$  for every  $\alpha \in k$ , infinitely many of the  $X_{\alpha}$  occur in one dimension. By virtue of  $\text{cx}_{\mathcal{G}}(M) = \dim \mathcal{V}_{\mathcal{G}}(X_{\alpha}) \geq 2$ , our observations above guarantee that these modules do not belong to homogeneous tubes of the stable Auslander-Reiten quiver of  $\mathcal{B}$ . Consequently, [6, Theorem D] implies the wildness of the block  $\mathcal{B}$ .  $\square$

We record the following consequence.

**Corollary 3.2.** *Let  $\mathcal{B} \subset M(\mathcal{G})$  be a tame block. Then we have*

$$\text{cx}_{\mathcal{G}}(M) \leq 2$$

for every  $\mathcal{B}$ -module  $M$ .

*Proof.* Suppose there exists a  $\mathcal{B}$ -module  $M$  of complexity  $\geq 3$ . Then we have

$$3 \leq \text{cx}_{\mathcal{G}}(M) = \dim \mathcal{V}_{\mathcal{G}}(M) \leq \dim \mathcal{V}_{\mathcal{B}},$$

so that (3.1) implies the wildness of  $\mathcal{B}$ .  $\square$

The corresponding result for representation-finite blocks of  $M(\mathcal{G})$  is considerably easier to establish and actually holds for arbitrary self-injective algebras. In view of [19], the non-projective indecomposable modules of a representation-finite self-injective algebra satisfy  $\Omega^n(M) \cong M$ , so that  $\text{cx}(M) \leq 1$ .

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