Self-organizing birth-and-death stochastic systems in continuum

Yuri Kondratiev,*
Robert Minlos and Elena Zhizhina †

Abstract

We consider birth-and-death stochastic particle systems in continuum which are under a self-regulation mechanism controlling configurations of particles via a pairwise interaction between them. The latter is reflected in a potential perturbation of the free generator. We show that the ground state renormalization scheme in the considered model leads to an invariant measure, a renormalized generator and resulting equilibrium birth-and-death stochastic dynamics for the system. The proof is based on the Gibbs type representation for related path space measure. This measure has OS-positivity property and is constructed via the cluster expansion method.

1 Introduction

We consider an infinite system of particles in continuum under a stochastic evolution corresponding to a heuristic generator

$$H \;=\; L_0 - \alpha U. \tag{1}$$

Here L_0 is the generator of a non-interacting birth-and-death process (a Glauber type dynamics), and U is an operator of multiplication by a function (equals to a sum of pair interactions over the configuration of points of

*Department of Mathematics, Bielefeld University, 33615 Bielefeld; kondrat@mathematik.uni-bielefeld.de

†Institute for Information Transmission Problems, Bolshoy Karetny per. 19, 127994 Moscow; minl@iitp.ru (R.Minlos), ejj@iitp.ru (E.Zhizhina)

the system), $0 < \alpha \ll 1$ is a coupling constant. The goal of the paper is to construct and to study a process properly associated with operator (1). We will use here an approach similar to used for the investigation of an infinite system of quantum anharmonic oscillators [10] or the quantum Heisenberg model [2]. Our approach is based on the Feynman-Kac formula:

$$e^{tH}(y_1, y_2) \ = \int\limits_{x_0=y_1}^{x_t=y_2} e^{-\alpha \int_0^t U(x_s)ds} \ d\mathcal{P}_z^0(x), \tag{2}$$

where the integration in (2) is over the distribution $\mathcal{P}_z^0$ on a space of trajectories $x = \{x_s, \ s \in R^1\}$ of a Glauber type free stochastic dynamics with the generator L_0 (so-called Surgailis process, see [21]). A rigorous meaning this formula has only under some regularity assumptions on the potential U. The expression in the right hand side of (2) up to a multiplicative constant coincides with a Gibbs reconstruction of the measure $\mathcal{P}_z^0$. The latter gives us a hope to apply well known methods from statistical physics to the construction and investigation of semigroup (2). Let us note that the Feynman-Kac formula is in common use for the study of models in quantum statistical physics and quantum field theory, when $\mathcal{P}_z^0$ is a measure on trajectories of a free process, usually defined by a Schrödinger operator, see [6, 19].

We briefly describe now our constructions and state main results of the paper. Initially, we consider truncated (over the space) potential U_Λ, where $\Lambda \subset \mathbb{R}^d$ is a bounded domain of the space $\mathbb{R}^d$. It means that we consider a system where particles interact only if they are inside of domain Λ. Then the operator $H_\Lambda \ = \ L_0 - \alpha U_\Lambda$ is defined correctly and it is unitary equivalent up to an additive constant to the generator of a stationary Markov process

$$\mathcal{G}_\Lambda = \{\gamma_t, \ t \in \mathbb{R}^1\}, \quad \gamma_t \in \Gamma$$

with values in a space Γ of locally finite configurations in $\mathbb{R}^d$. The path space measure of the process $\mathcal{G}_\Lambda$ may be obtained as the limit when $T \to \infty$ of the Gibbs reconstructions by the energy function $\int_{-T}^{T} U_\Lambda(\gamma_s)ds$ of the reference measure corresponding to the Surgailis process [21] with the generator L_0. Then taking the thermodynamic limit as $\Lambda \nearrow \mathbb{R}^d$ we get the limit path space measure and the stochastic process $\mathcal{G}_\infty$. We prove that the limit process meets the condition of OS-positivity (Osterwalder-Schrader positivity, see, e.g., [18]). Using that fact we can construct in canonical way corresponding Hilbert space $\mathcal{H}$ and a semigroup of self-adjoint operators in $\mathcal{H}$ associated with the process $\mathcal{G}_\infty$ and generated by an operator $\hat{H}$. Thus through the use of the operator $\hat{H}$ heuristic expression (1) gains rigorous meaning, and $\hat{H}$ should be considered as a correct regularization of the operator (1). In addition, we prove the existence of the spectral gap for the operator $\hat{H}$ using estimates on decay of correlations for the limit process $\mathcal{G}_\infty$. The main technique we use here is based on cluster expansion methods for point fields developed in [12, 14, 9, 5].

Let us discuss possible interpretations of our results in individual based models of spatial economics. In this case, a configuration should be considered as a set of economic units (points of the configuration) located in the space. A pure birth Markov process corresponds to an economic growth model in which the density of units is linearly growing in time. Assuming additionally random life time of any unit (independent and exponentially distributed for each existing one), we will arrive in the Surgailis process mentioned above. The equilibrium measure of this process is Poisson one and its Markov generator is a self-adjoint operator in corresponding Poisson L^2-space, see [21]. This generator admits a nice and easy spectral decomposition that relates one to the Fock space number operator in quantum field theory, see, e.g. [1]. Actually, the Surgailis process can also be considered as a free Glauber type stochastic dynamics in continuum, see, e.g. [8]. In a more realistic model, we should take into account a competition between units. One way to include this notion is related with a modification of the death rate in the generator s.t. the growing density of the configuration will increase the intensity of death.

Another possibility is based on the consideration of a rate functional which should play the role of a regulation mechanism in the economic society. Namely, configurations of units with high rate must have less chances to survive in the stochastic evolution of the system. This rate functional is included as the potential U in the model considered in this paper. Main question which appears here is the existence of an equilibrium state in such economic models as well as the construction of related equilibrium stochastic process of the economic development with described regulation based on a local interaction between units. The results of the present paper give a positive answer to this problem.

2 The model and main results

2.1 Free Glauber dynamics (Surgailis process)

The configuration space $\Gamma = \Gamma(\mathbb{R}^d)$ of the model is the set of all locally finite subsets of $\mathbb{R}^d$: $\gamma \subset \mathbb{R}^d$. The space Γ is naturally endowed with a topology, namely the weakest topology on Γ with respect to which all maps $\Gamma \ni \gamma \mapsto \langle f, \gamma \rangle := \sum_{x\in\gamma} f(x)$, $f \in C_0(\mathbb{R}^d)$, are continuous (here, $C_0(\mathbb{R}^d)$ is the space of all continuous real-valued functions on $\mathbb{R}^d$ with compact support). We denote by $\mathcal{B}(\Gamma)$ the Borel σ-algebra on Γ generated by this topology, and let π_z be the Poisson measure on $(\Gamma,\ \mathcal{B}(\Gamma))$ with activity $z,\ z > 0$, see [7]. We define a stationary Markov process on Γ with the invariant measure π_z. A generator of the corresponding stochastic semigroup S_t^0 acting in the functional space $L_2(\Gamma, d\pi_z)$ has a form

$$(L_0 F)(\gamma) = \sum_{x\in\gamma} (F(\gamma \setminus x) - F(\gamma)) + z \int_{\mathbb{R}^d} (F(\gamma \cup x) - F(\gamma))\, dx. \tag{3}$$

The operator L_0 is defined on local bounded functions $F(\gamma)$, and the expression (3) can be extended to a self-adjoint operator in $L_2(\Gamma, d\pi_z)$. The corresponding process is a birth-and-death process on Γ, we also call it the free Glauber dynamics or the Surgailis process, see [21]. The process can be described as follows: each particle in the configuration can disappear after an exponentially distributed life time and new particles can appear in the configuration with intensity z uniformly over the space. We denote by $\mathcal{P}_z^0$ a distribution of the Surgailis process (on the trajectory space).

2.2 Glauber dynamics with interaction

The generator of the dynamics with interaction is given heuristically as follows:

$$(HF)(\gamma) \ = \ (L_0 F)(\gamma) - \alpha U(\gamma) F(\gamma) \tag{4}$$

where

$$U(\gamma) = \sum_{\{x,y\}\subset\gamma} \varphi(x - y). \tag{5}$$

Here $\varphi(u) \geq 0,\ u \in \mathbb{R}^d$ is an even real-valued function (a potential) with a fast decreasing on the infinity (we give the precise conditions on φ below),

$\alpha > 0$ is a small enough real constant.

Remark. We consider a non-negative potential φ only for simplification of our reasoning below. Let us stress that all results and constructions of the present paper are true for any general stable potential, see [17], with a fast decreasing on the infinity.

We will introduce below a dynamics with the generator (4) as a limit of dynamics given in bounded regions $\Lambda \subset \mathbb{R}^d$. Namely, let us consider the operator

$$(H_\Lambda F)(\gamma) \;=\; (L_0 F)(\gamma) - \alpha U_\Lambda(\gamma) F(\gamma), \quad F \in L_2(\Gamma, d\pi_z) \tag{6}$$

with

$$U_\Lambda(\gamma) = \sum_{\{x,y\}\subset\gamma\cap\Lambda} \varphi(x-y). \tag{7}$$

Theorem 1. *Under assumptions (55) - (56) on the function φ and small enough α we have for all bounded $\Lambda \subset \mathbb{R}^d$:*

1. the operator H_Λ is selfadjoint and bounded from above;

2. a non-degenerate ground state of H_Λ exists, i.e. a unique normalized eigenvector Ψ_Λ of the operator H_Λ such that $\Psi_\Lambda(\gamma) > 0$ exists, and corresponding eigenvalue λ^0_Λ is the same as the upper boundary of the spectrum of H_Λ.

Proof of the theorem see in Sect 6.

We apply below the general scheme of the ground state transformation for potential perturbations of Markov generators, see e.g. [3]. Assign a new measure on Γ in the following way:

$$\frac{d\nu_z^\Lambda}{d\pi_z}(\gamma) = \left(\Psi_\Lambda(\gamma)\right)^2. \tag{8}$$

Define an unitary transformation

$$W_\Lambda : L_2(\Gamma, \nu_z^\Lambda) \mapsto L_2(\Gamma, \pi_z):\ (W_\Lambda F)(\gamma) = \Psi_\Lambda(\gamma) F(\gamma),\ F \in L_2(\Gamma, \nu_z^\Lambda) \tag{9}$$

and the operator $\tilde{H}_\Lambda$ in $L_2(\Gamma, \nu_z^\Lambda)$

$$\tilde{H}_\Lambda = W_\Lambda^{-1}(H_\Lambda - \lambda^0_\Lambda I) W_\Lambda \tag{10}$$

which is unitary equivalent to $H_\Lambda - \lambda_\Lambda^0 I$, where I is the identity operator in $L_2(\Gamma, \pi_z)$. It follows from (3) and (6) that operator $\tilde{H}_\Lambda$ has the following form:

$$(\tilde{H}_\Lambda F)(\gamma) \;=\; \sum_{x\in\gamma} \frac{\Psi_\Lambda(\gamma\backslash x)}{\Psi_\Lambda(\gamma)}\,(F(\gamma\backslash x) - F(\gamma)) \;+$$

$$z\int_{R^d} \frac{\Psi_\Lambda(\gamma\cup y)}{\Psi_\Lambda(\gamma)}\,(F(\gamma\cup y) - F(\gamma))\,dy.$$

Clear that

$$\tilde{H}_\Lambda \mathbf{1} \;=\; 0, \tag{11}$$

where $\mathbf{1} \in L_2(\Gamma, \nu_z^\Lambda)$ is the constant function equals to 1. We denote by

$$S_t^\Lambda \;=\; \exp\{tH_\Lambda\} \quad \text{and} \quad \tilde{S}_t^\Lambda \;=\; \exp\{t\tilde{H}_\Lambda\} \tag{12}$$

semigroups acting in the spaces $L_2(\Gamma, \pi_z)$ and $L_2(\Gamma, \nu_z^\Lambda)$ correspondingly. We have the following representation for the kernel of the semigroup e^{tH_Λ} using the Feynman-Kac formula

$$S_t^\Lambda(\gamma_1,\gamma_2) \;=\; R(\gamma_1,\gamma_2) \;=\; \int_{\substack{\gamma(0)=\gamma_1\\ \gamma(t)=\gamma_2}} \exp\left\{-\alpha\int_0^t U_\Lambda(\gamma(\tau))d\tau\right\} d\mathcal{P}_z^0(\gamma), \tag{13}$$

where $\mathcal{P}_z^0$ is the distribution of the Surgailis process. It follows from representation (13) and the strict positivity of Ψ_Λ that semigroups (12) improve positivity, see [16]. Moreover, relation (11) implies that

$$\exp\{t\tilde{H}_\Lambda\}\;1 \;=\; 1. \tag{14}$$

Consequently, $\tilde{S}_t^\Lambda$ is the Markov semigroup and the associated process is a Markov stationary process with the invariant measure ν_z^Λ. Thus, the following theorem holds.

Theorem 2. *Semigroup* $\tilde{S}_t^\Lambda \;=\; \exp\{t\tilde{H}_\Lambda\}$ *is the Markov semigroup. The process*

$$\mathcal{G}_\Lambda = \{\gamma_t, \quad t\in R^1\}, \quad \gamma_t \in \Gamma \tag{15}$$

associated with the semigroup $\tilde{S}_t^\Lambda$ *is the stationary reversible Markov process on* Γ *with the invariant measure* ν_z^Λ.

We denote by $\mathcal{P}_{\Lambda,z}$ a distribution of the process $\mathcal{G}_\Lambda$. As any stationary reversible Markov process, process (15) has property of OS-positivity. We remind this notion and also some related facts, see [18]. Let $x = \{x_t,\ t \in R^1\}$ be a stationary reversible process on the space X and $\mathcal{P}$ be a distribution on trajectories of the process. Introduce the time reflection transform ϑ in the space of trajectories $\Omega = X^R$:

$$(\vartheta\ x)_t = x_{-t}, \quad x \in \Omega. \tag{16}$$

Since the process is reversible, ϑ preserves the distribution $\mathcal{P}$. The unitary representation for ϑ in the space $L_2(\Omega, \mathcal{P})$ we denote θ:

$$(\theta f)(x) = f(\vartheta x), \quad f \in L_2(\Omega, \mathcal{P}),\ x \in \Omega.$$

Let $\mathcal{H}_+ \subset L_2(\Omega, \mathcal{P})$ be a subspace of functions on Ω depending on the process "at present and in future":

$$f \in \mathcal{H}_+ : \quad f(x) = f(\{x_t\},\ t \in [0, \infty)).$$

Then the reversible process x called OS-positive if for any $f \in \mathcal{H}_+$ the quadratic form

$$(\theta f, f)_{L_2(\Omega,\mathcal{P})} \ \equiv\ (f, f)_{\mathcal{H}_+} \ =\ (f, f)_+ \ \geq\ 0. \tag{17}$$

is non-negative. We notice that any Markov stationary reversible process is always OS-positive since

$$(f, f)_+ \ =\ \|P(\mathcal{H}_0)f\|^2, \quad f \in \mathcal{H}_+,$$

where $P(\mathcal{H}_0)$ is a projection to the space $\mathcal{H}_0$ of functions depending only on values of the process x_0 at zero time.

For the stationary reversible OS-positive process x we can construct a semigroup which is similar to the stochastic semigroup for a Markov process. If $I_0 \subset \mathcal{H}_+$ is the kernel of the quadratic form (17):

$$I_0 = \{f \in \mathcal{H}_+ :\ (f, f)_+ \ =\ 0\},$$

then I_0 is a closed subspace of $\mathcal{H}_+$ and we can consider a factor-space $G = \mathcal{H}_+/I_0$. The scalar product in G is defined by the following way

$$([f_1], [f_2])_G \ =\ (f_1, f_2)_+, \tag{18}$$

where $[f] \in G$ is the class of the element $f \in \mathcal{H}_+$. The space G is usually called the physical space of the process x. We denote by U_t a unitary operator of a time shift acting in $L_2(\Omega, \mathcal{P})$ in the following way

$$(U_t f)(x) = f(s_{-t}x),$$

where s_τ are shifts in the space of trajectories

$$(s_\tau x)_t = x_{t-\tau}.$$

Clear, $U_t\mathcal{H}_+ \subset \mathcal{H}_+$ for any $t > 0$. Then the permutation relation

$$\theta U_t = U_{-t}\theta$$

together with the unitarity of U_t in $L_2(\Omega, \mathcal{P})$ imply that the operators $U_t,\ t > 0$ are symmetrical with respect to the quadratic form (17):

$$(\theta U_t f_1, f_2)_{L_2(\Omega,\mathcal{P})} \;=\; (\theta f_1, U_t f_2)_{L_2(\Omega,\mathcal{P})}, \quad f_1, f_2 \in \mathcal{H}_+. \tag{19}$$

Proposition 1. *For any $f \in \mathcal{H}_+$ and $t \geq 0$ the following inequality holds*

$$(U_t f, U_t f)_+ \;\leq\; (f, f)_+. \tag{20}$$

Proof see in Attachment.

Inequality (20) implies that $U_t I_0 \subset I_0$ for $t > 0$, consequently the semigroup $\{U_t,\ t \geq 0\}$ in $\mathcal{H}_+$ generates the semigroup $\hat{U}_t,\ t \geq 0$ of operators in G:

$$\hat{U}_t[f] = [U_t f], \quad t > 0,\ f \in \mathcal{H}_+.$$

It follows from (19)-(20) that $\hat{U}_t$ is a selfadjoint contraction semigroup. In addition, it is strongly-continuous by construction. This semigroup $\hat{U}_t$ is called a tansfer-matrix of the process x. The Stone theorem, see e.g. [16], implies that the operators $\hat{U}_t$ have the form:

$$\hat{U}_t = e^{th},$$

where h is a non-positive selfadjoint operator in G. Note, that the element $e = [1]$ is a normalized ground state of the operator h with the eigenvalue 0.

Let F be a local function on the space of trajectories $\{\gamma_t,\ t \in R\}$ of the process, that means there exist a finite interval $I \subset R^1$ and a bounded domain $\Lambda_0 \subset R^d$ such that the function F depends only on $\{\gamma_t \cap \Lambda_0,\ t \in I\}$, i.e. on the part of trajectories $\{\gamma_t,\ t \in R\}$ lying inside of a bounded domain $\Lambda_0 \times I = M$ in R^{d+1}. The domain $M = \Lambda_0 \times I$ is called the localization domain for F. Remind that the weak convergence of the processes $\mathcal{P}_{\Lambda,z} \Rightarrow \mathcal{P}_{\infty,z}$ means that for any local bounded function F the following holds as $\Lambda \nearrow R^d$

$$\langle F \rangle_{\mathcal{P}_{\Lambda,z}} \mapsto \langle F \rangle_{\mathcal{P}_{\infty,z}}, \tag{21}$$

where $\langle \cdot \rangle_{\mathcal{P}_{\Lambda,z}}$ means the average over the distribution $\mathcal{P}_{\Lambda,z}$ and the same for $\langle \cdot \rangle_{\mathcal{P}_{\infty,z}}$.

Theorem 3. *Under conditions of Theorem 1 distributions $\mathcal{P}_{\Lambda,z}$ of the processes (15) converge weakly as $\Lambda \nearrow R^d$ to the distribution $\mathcal{P}_{\infty,z}$ of a stationary reversible OS-positive process*

$$\mathcal{G}_\infty = \{\gamma_t, \quad t \in R^1\} \tag{22}$$

with values $\gamma_t \in \Gamma$. Moreover, the stationary distributions ν_z^Λ converge weakly to the marginal distribution ν_z^∞ of the process (22).

The weak convergence of measures on the space Γ of locally-finite configurations is defined by the same way as in (21) with local functions F depending on the part of the configuration $\gamma \in \Gamma$ in a bounded domain $\Lambda_0 \subset R^d :\ F(\gamma) = F(\gamma|_{\Lambda_0})$.

Theorem 3 is a corollary of Theorem 4 below, a construction and investigations of process (22) will be done in the proof of Theorem 4.

Remark. Using as above OS-positivity of the process (22) with the distribution $\mathcal{P}_{\infty,z}$ we can introduce the generator of its transfer-matrix $h = \hat{H}$ acting in the corresponding physical space G. The generator can be treated as a correctly defined limit Hamiltonian (up to an additive constant) associated to the formal Hamiltonian (4). The operator $\hat{H}$ can be considered as a regularized limit for the operators $\hat{H}_\Lambda$.

Conjecture. Although we proved here only that the limit process has the property of OS-positivity, we believe that the limit process should be Markov.

3 Euclidean representation

3.1 Path space measure for free Glauber dynamics

We denote by Υ a space of configurations

$$\eta = (\gamma, l_\gamma) = \{(x, l_x) \mid x \in \gamma\}$$

of a marked point field in the space $R^d \times R^1 = R^{d+1}$. Here $\gamma \subset R^{d+1}$ is a locally-finite configuration of points in R^{d+1}:

$$x \in \gamma : \; x = (s, t) \in R^{d+1}, \quad s \in R^d, \; t \in R^1,$$

and $l_x \in (0, \infty)$ is a value of the mark at the point x. The distribution $\mathcal{P}_z^0$ of the marked point field can be described by the following way: point configurations γ form the Poisson field in R^{d+1} with an intensity $z > 0$ (the corresponding distribution is denoted by Π_z, see [7]), and under a fixed point configuration γ the conditional distribution of marks is conditionally independent and exponential:

$$\Pr(l > u) = e^{-u}, \; u \geq 0. \tag{23}$$

A configuration $\eta \in \Upsilon$ can be visually depicted as a configuration η of rods $\xi \in \eta$ lying in R^{d+1} and directed along the positive direction of the time axis t. Here $x = x(\xi) = (s, t) \in R^{d+1}$ is the origin of the rod, and $l = l(\xi)$ is the length of the rod. Let K be a space of all rods in R^{d+1} lying along a given (time) direction, then K is the same as $R^{d+1} \times R^1_+$, and the described above configuration space Υ of rods could be identified with a subset of all locally finite configurations $\Gamma(R^{d+1} \times R^1_+)$ of points in $R^{d+1} \times R^1_+$. This permits to introduce a topology and the Borel σ-algebra on Υ generated by this topology.

We shall say that a configuration of rods η is locally finite if any bounded subset of R^{d+1} has an intersection only with a finite number of rods from this configuration. Let us denote by $\Upsilon' \subset \Upsilon$ a set of locally finite configurations composed of pairwise disjoint rods.

Lemma 1. *The set Υ' of locally finite configurations of pairwise disjoint rods form a set of the full measure $\mathcal{P}_z^0$.*

For the proof, see Attachment.

For any configuration $\eta \in \Upsilon'$ and any $\tau \in R^1$ we consider a section $\eta \cap Y_\tau \subset R^{d+1}$ of rods from the configuration η by the hyperplane $Y_\tau = \{x = (s,t) : t = \tau\}$. Then by Lemma 1 we have that the projection of the section to the space R^d is a locally finite set $\gamma_\tau \in \Gamma(R^d)$. Thus, any configuration of rods $\eta \in \Upsilon'$ generates a curve $\gamma = \{\gamma_\tau,\ \tau \in R\}$ in the space $\Gamma(R^d)$, and different curves correspond to different configurations of rods. Let a set of these curves will be Σ. Then the distribution $\mathcal{P}_z^0$ can be regarded as a distribution on Σ, in this case call it $\hat{\mathcal{P}}_z^0$. Thus, using Lemma 1 we get the following

Lemma 2. *The above curves $\gamma = \{\gamma_\tau,\ \tau \in R\} \in \Sigma$ form the full measure set of trajectories of the free Glauber dynamics from Sect. 2.1 with the generator (3), and the distribution $\hat{\mathcal{P}}_z^0$ on Σ is the same as the distribution of the Glauber dynamics.*

In the representation for the trajectory $\gamma = \{\gamma_\tau,\ \tau \in R\}$ in the form of a rod configuration η the origin $x = (s,t)$ of the rod $\xi \in \eta,\ \xi = (x, l_x)$ marks the position s and the time t of the birth of a new particle in the point configuration, and the length l_x is the life time of the particle.

Remark. Reversibility of the free Glauber dynamics implies that the above field of rods is also reversible in time. Indeed, ends of rods under reflection in time come to origins of the reflected rods, but the point field corresponding to the ends of all rods is also the Poisson field in R^{d+1} with the intensity z. This fact has been discussed earlier, see for instance [4].

3.2 Euclidean representation for dynamics of interacting particles (ensemble of rods)

For any bounded $\Lambda \subset R^d$ and any $0 < T < \infty$ we consider a new probability measure $\hat{\mathcal{P}}_{\Lambda,T,z}$ on Σ using Feynmann-Kac representation:

$$\frac{d\hat{\mathcal{P}}_{\Lambda,T,z}}{d\hat{\mathcal{P}}_z^0}(\gamma) = \frac{1}{Z_{\Lambda,T}} \exp\left\{-\alpha \int_{-T}^{T} U_\Lambda(\gamma_\tau) d\tau\right\},\ \gamma = \{\gamma_\tau,\ \tau \in R\} \tag{24}$$

with the normalization factor

$$Z_{\Lambda,T} = \int\limits_{\Sigma} \exp\left\{-\alpha \int_{-T}^{T} U_\Lambda(\gamma_\tau) d\tau\right\} d\hat{\mathcal{P}}_z^0. \tag{25}$$

Since $U_\Lambda(\gamma) > 0$ and $\int_{-T}^{T} U_\Lambda(\gamma_\tau)\, d\tau < \infty$ on a set of the full measure, then $0 < Z_{\Lambda,T} < \infty$ and relation (24) is correctly defined.

We denote by $\mathcal{P}_{\Lambda,T,z}$ a measure on the configuration space of rods which is corresponding to $\hat{\mathcal{P}}_{\Lambda,T,z}$. Then the probability density (24) is rewritten as

$$\frac{d\mathcal{P}_{\Lambda,T,z}}{d\mathcal{P}_z^0}(\eta) = \frac{1}{Z_{\Lambda,T}} \exp\left\{ -\alpha \sum_{\{\xi_{j_1},\ \xi_{j_2}\}\subset\eta_{\Lambda,T}} \Phi^T(\xi_{j_1},\ \xi_{j_2}) \right\}, \tag{26}$$

where $\eta_{\Lambda,T} \subseteq \eta$ is a subset of rods from configuration η which have intersection with $\Lambda \times [-T,T] \subset R^{d+1}$, and

$$\Phi^T(\xi_1,\xi_2) = \varphi(s_1 - s_2)\ \Delta^T(\xi_1,\xi_2), \tag{27}$$

with $\xi_i = (\ (s_i,t_i),\ l_i\ ),\ \ i = 1,2$, and

$$\Delta^T(\xi_1,\xi_2) = |(t_1, t_1 + l_1) \cap (t_2, t_2 + l_2) \cap (-T,T)| \tag{28}$$

is a length of the common part of the projections to the axis t of the rods ξ_1 and ξ_2 which are inside of $[-T,T]$. We introduce the following notation

$$\Delta(\xi_1,\xi_2) = \lim_{T\to\infty} \Delta^T(\xi_1,\xi_2) = |(t_1, t_1 + l_1) \cap (t_2, t_2 + l_2)|. \tag{29}$$

Thus the measure $\mathcal{P}_{\Lambda,T,z}$ on the space Υ' of rods is the Gibbs reconstruction of the measure $\mathcal{P}_z^0$ by means of the following pair interaction

$$U_{\Lambda,T}(\eta) = \sum_{\{\xi_1,\ \xi_2\}\subset\eta_{\Lambda,T}} \Phi^T(\xi_1,\xi_2)$$

For any $M \subset R^{d+1}$ we consider a set G_M^{int} of all rods intersecting M and a set $G_M^{loc} \subset G_M^{int}$ of all rods with origins in M. We say that a set of rods $G \subset K$ is bounded if there exists a bounded set $M \subset R^{d+1}$ such that $G \subseteq G_M^{int}$, and is strictly bounded if $G \subseteq G_M^{loc}$. For any $G \subset K$ let $\Upsilon'(G) \subseteq \Upsilon'$ be a set of all locally finite configurations of pairwise disjoint rods from G. In the case of bounded G the space $\Upsilon'(G)$ contains finite configurations η. For any $G \subset K$ we can represent configurations $\eta \in \Upsilon'$ as

$$\eta = (\eta_G, \eta_{G'}),\ \eta_G \in \Upsilon'(G),\ \eta_{G'} \in \Upsilon'(G') \quad \text{with} \quad G' = K \setminus G.$$

This representaion implies the following decomposition of Υ' to the Cartesian product

$$\Upsilon' = \Upsilon'(G) \times \Upsilon'(G').$$

We say that a function $F = F(\eta)$ defined on Υ' is local if there exists a bounded set $M \subset R^{d+1}$ such that F depends only on $\eta_{G_M^{int}} \subseteq \eta : \; F(\eta) = F(\eta_{G_M^{int}})$. If a function F depends only on $\eta_{G_M^{loc}} \subseteq \eta : F(\eta) = F(\eta_{G_M^{loc}})$ we denote the function F strongly local. In any case, the set $M \subset R^{d+1}$ is called the localization domain of the function F. Let us note, that each local function on the space of trajectories Σ with the localization domain $\Lambda_0 \times I = M$ regarded as a function on the configuration space of rods Υ' with the localization domain M.

Theorem 4. *We assume that $\varphi \geq 0$ is a non-negative integrable function and α is a small enough. Then*

1. the distributions $\mathcal{P}_{\Lambda,T,z}$ converge weakly as $T \nearrow \infty$ to the distribution $\mathcal{P}_{\Lambda,\infty,z} = \mathcal{P}_{\Lambda,z}$

$$\mathcal{P}_{\Lambda,\infty,z} = w - \lim_{T\to\infty} \mathcal{P}_{\Lambda,T,z}, \tag{30}$$

and the corresponding distribution $\hat{\mathcal{P}}_{\Lambda,\infty,z} = \hat{\mathcal{P}}_{\Lambda,z}$ on the space Σ is a distribution of a stationary reversible Markov process on Γ

$$\mathcal{G}_\Lambda = \{\gamma_t, \quad t \in R^1\} \tag{31}$$

with the invariant measure ν_z^Λ and associated stochastic semigroup $\tilde{S}_t^\Lambda$ generated by $\tilde{H}_\Lambda$. Thus, process (31) is exactly the same as process (15) from Theorem 2, that has been constructed by the different way.

2. There exists a weak limit

$$\mathcal{P}_{\infty,z} = w - \lim_{\Lambda\nearrow R^d} \mathcal{P}_{\Lambda,\infty,z} = w - \lim_{T\to\infty,\ \Lambda\nearrow R^d} \mathcal{P}_{\Lambda,T,z}. \tag{32}$$

The corresponding distribution $\hat{\mathcal{P}}_{\infty,z}$ on the space of trajectories Σ is a distribution of a stationary reversible OS-positive process

$$\mathcal{G}_\infty = \{\gamma_t, \quad t \in R^1\} \tag{33}$$

with values $\gamma_t \in \Gamma$ and the marginal distribution $\nu_z^\infty = w - \lim_{\Lambda\nearrow R^d} \nu_z^\Lambda$. Thus, the process (33) is the same as limit process (22) from theorem 3.

Here as above the weak convergence of distributions means the convergence of averages over corresponding distributions for any bounded local function F defined on Υ'.

Remark As we have already mentioned above, all statements of Theorem 4 remain valid in the case of a stable integrable potential φ (not necessarily, nonnegative), although corresponding reasonings in the proof require some evident modifications.

We formulate next results on decay of correlations for the distributions $\mathcal{P}_{\Lambda,z}$ and the limit distribution $\mathcal{P}_{\infty,z}$. We assume that for all large enough $|u|$ the potential φ meets one of the folowing estimates:

$$1)\ |\varphi(u)|\ <\ \frac{c}{(1+|u|)^{2m}},\ m>d, \quad 2)\ |\varphi(u)|\ <\ ce^{-k|u|},$$

with constants $c>0,\ m>d,\ k>0$, and introduce the following metrics in the space $R^d\times R^1$:

$$\varrho((s_1,t_1),(s_2,t_2)) = \begin{cases} m\ln(1+|s_1-s_2|)+\frac{1}{2}|t_1-t_2|, & \text{in the case } 1), \\ \\ \frac{k}{2}|s_1-s_2|+\frac{1}{2}|t_1-t_2|, & \text{in the case } 2) \end{cases} \tag{34}$$

with $s_1,s_2\in R^d,\ t_1,t_2\in R^1$.

Theorem 5. *For any strongly local bounded functions $F_1,\ F_2$ depending on the process $\mathcal{G}_\infty$ (or $\mathcal{G}_\Lambda$) with localization domains $M_i=\Lambda_i\times I_i,\ i=1,2$ correspondingly the following estimate holds:*

$$\left|\langle F_1\cdot F_2\rangle_{\mathcal{P}_{\infty,z}}-\langle F_1\rangle_{\mathcal{P}_{\infty,z}}\langle F_2\rangle_{\mathcal{P}_{\infty,z}}\right| < C^{|M_1|+|M_2|}\ (|M_1|+|M_2|)\ e^{-d(M_1,M_2)}, \tag{35}$$

and the analogous one is true for the processes $\mathcal{G}_\Lambda$. Here $C>0$ is a constant, $|M_i|$ are $d+1$-dimensional volumes of the domains M_i, and $d(M_1,M_2)$ is the distance between the domains in the metric (34):

$$d(M_1,M_2)=\inf_{(s_i,t_i)\in M_i,i=1,2}\varrho((s_1,t_1),(s_2,t_2)).$$

Corollary. The operator $\hat{H}$ has a spectral gap, i.e. there exists a gap between 0 and the spectrum of $\hat{H}$ in the orthogonal complement to the ground state $e \in G$. In particular, that implies the uniqueness of the ground state.

For computational convenience we modify the definition of the measure $\mathcal{P}_{\Lambda,T,z}$. We call $G^0_{\Lambda,T} = G^{int}_{\Lambda\times(-T,T)} \subset K$ a set of rods which have an intersection with $\Lambda \times (-T, T) \subset R^{d+1}$, $G^{\pm}_{\Lambda,T} \subset K$ a set of rods entirely belonging to the region $\Lambda \times (T, +\infty)$ (in the case $+$) and correspondingly, entirely belonging to the region $\Lambda \times (-\infty, -T)$ (in the case $-$), $G_{\Lambda',\infty} \subset K$ a set of rods lying inside $\Lambda' \times (-\infty, \infty) \subset R^{d+1}$ with $\Lambda' = R^d \backslash \Lambda$. Obviously, these sets are mutually disjoint and their union is the same as K. Then any configuration $\eta \in \Upsilon'$ is the sum of 4 mutually disjoint configurations

$$\eta \; = \; \eta_{G^0_{\Lambda,T}} \cup \eta_{G^+_{\Lambda,T}} \cup \eta_{G^-_{\Lambda,T}} \cup \eta_{G_{\Lambda',\infty}}, \tag{36}$$

and the configuration space Υ' is the Cartesian product of the spaces

$$\Upsilon' \; = \; \Upsilon'(G^0_{\Lambda,T}) \times \Upsilon'(G^+_{\Lambda,T}) \times \Upsilon'(G^-_{\Lambda,T}) \times \Upsilon'(G_{\Lambda',\infty}). \tag{37}$$

We denote by

$$\mathcal{P}^0_{G^0_{\Lambda,T},z},\; \mathcal{P}^0_{G^+_{\Lambda,T},z},\; \mathcal{P}^0_{G^-_{\Lambda,T},z},\; \mathcal{P}^0_{G_{\Lambda',\infty},z} \tag{38}$$

distributions on the spaces of corresponding configurations, i.e. restrictions of the distribution $\mathcal{P}^0_z$ to the sets

$$\Upsilon'(G^0_{\Lambda,T}),\; \Upsilon'(G^+_{\Lambda,T}),\; \Upsilon'(G^-_{\Lambda,T}),\; \Upsilon'(G_{\Lambda',\infty})$$

respectively.

We consider next a general random field $\Pi(M, \zeta, p)$. Here $M \subset R^{d+1}$ is a domain in R^{d+1}, $\zeta = \zeta(x)$, $x \in M$ is a positive function defined on M, the function $\zeta(x)$ specifies an activity (non-homogeneous, in general) of the Poisson field of the rods origins $x \in M$, $p = \{p_x(l) = \Pr(l_x > l),\ l > 0,\ x \in M\}$ is a family of distribution functions marked by points $x \in M$ for the length of a rod with the origin at $x \in M$. Under fixed origins of the rods their lengths have conditionally-independent distributions with densities $p_x(l)$.

Lemma 3. *1. All components $\eta_{G^0_{\Lambda,T}}$, $\eta_{G^+_{\Lambda,T}}$, $\eta_{G^-_{\Lambda,T}}$, $\eta_{G_{\Lambda',\infty}}$ in decomposition (36) are independent, i.e.*

$$\mathcal{P}^0_z \; = \; \mathcal{P}^0_{G^0_{\Lambda,T},z} \times \mathcal{P}^0_{G^+_{\Lambda,T},z} \times \mathcal{P}^0_{G^-_{\Lambda,T},z} \times \mathcal{P}^0_{G_{\Lambda',\infty},z}. \tag{39}$$

2. Each distribution from (38) is a distribution of the form $\Pi(M,\zeta,p)$*, namely,*
a) in the case $\mathcal{P}^0_{G_{\Lambda',\infty},z}$

$$M=\Lambda'\times(-\infty,+\infty),\ \zeta(x)\equiv z,\ p_x(l)=e^{-l},\ l>0,\ x\in M; \tag{40}$$

b) in the case $\mathcal{P}^0_{G^+_{\Lambda,T},z}$

$$M=\Lambda\times(T,+\infty),\ \zeta(x)\equiv z,\ p_x(l)=e^{-l},\ l>0,\ x\in M; \tag{41}$$

c) in the case $\mathcal{P}^0_{G^0_{\Lambda,T},z}$

$$M=\Lambda\times(-\infty,T),\ \zeta(x)=\left\{\begin{array}{c} z,\ x\in\Lambda\times(-T,T) \\ ze^{-\tau},\ x\in\Lambda\times(-\infty,-T)\end{array}\right., \tag{42}$$

$$p_x(l)=\left\{\begin{array}{c} e^{-l},\ l>0,\ x\in\Lambda\times(-T,T) \\ e^{\tau}e^{-l}\ \chi(\tau,\infty)+\chi(0,\tau),\ x\in\Lambda\times(-\infty,-T)\end{array}\right.,$$

where $\tau=-T-t>0$ *is a distance from* $x=(s,t)\in\Lambda\times(-\infty,-T)$ *to the hyperplane* $Y_{-T}=\{x:t=-T\}$*,* $\chi(a,b)$ *is the characteristic function of the interval* (a,b)*;*
d) in the case $\mathcal{P}^0_{G^-_{\Lambda,T},z}$

$$M=\Lambda\times(-\infty,-T),\ \zeta(x)=z(1-e^{-\tau}),\ p_x(l)=e^{-l}\ \chi(0,\tau),\ l>0,\ x\in M. \tag{43}$$

Proof of Lemma 3. We consider a decomposition of a configuration $\eta\in\Upsilon'$ to four configurations

$$\eta\ =\ \hat{\eta}_{\Lambda,0}\cup\hat{\eta}_{\Lambda,+}\cup\hat{\eta}_{\Lambda,-}\cup\hat{\eta}_{\Lambda'}. \tag{44}$$

Here $\hat{\eta}_{\Lambda'}$ is a configuration of rods which are entirely outside of the cylinder $\Lambda\times(-\infty,+\infty)$, $\hat{\eta}_{\Lambda,-}$ is a configuration from rods with origins in $\Lambda\times(-\infty,-T)$, $\hat{\eta}_{\Lambda,0}$ is a configuration from rods with origins in $\Lambda\times(-T,T)$ and $\hat{\eta}_{\Lambda,+}$ is a configuration of rods which are entirely inside of the region $\Lambda\times(T,+\infty)$.

Since any rod $\xi\in\eta$ belongs to one of the sub-configurations

$$\hat{\eta}_{\Lambda,0},\ \hat{\eta}_{\Lambda,+},\ \hat{\eta}_{\Lambda,-},\ \hat{\eta}_{\Lambda'},$$

and it is determined only by the origin of the rod ξ, all these configurations are configurations of a marked Poisson field in the corresponding volumes $\Lambda \times (-T,T)$, $\Lambda \times (T,\infty)$, $\Lambda \times (-\infty,-T)$, $\Lambda' \times (-\infty,\infty)$. That implies independence of all configurations, and compareing (36) with (44) we have:

$$\hat{\eta}_{\Lambda'} = \eta_{G_{\Lambda',\infty}},\ \hat{\eta}_{\Lambda,+} = \eta_{G^{+}_{\Lambda,T}},\ \hat{\eta}_{\Lambda,0} \cup \hat{\eta}_{\Lambda,-} = \eta_{G^{0}_{\Lambda,T}} \cup \eta_{G^{-}_{\Lambda,T}}.$$

The configuration $\hat{\eta}_{\Lambda,-}$ can be decomposed into two configurations

$$\hat{\eta}_{\Lambda,-} = \breve{\eta}_{\Lambda,-} \cup \breve{\eta}_{\Lambda,0},$$

where $\breve{\eta}_{\Lambda,-}$ is a configuration of rods which are entirely inside of the region $\Lambda \times (-\infty,-T)$, $\breve{\eta}_{\Lambda,0}$ is a configuration of rods with origins in $\Lambda \times (-\infty,-T)$ which have intersection with $\Lambda \times (-T,T)$. Since the question about belonging a rod $\xi \in \hat{\eta}_{\Lambda,-}$ to one of the configurations $\breve{\eta}_{\Lambda,-}$, $\breve{\eta}_{\Lambda,0}$ depends on the length of the rod, the configurations $\breve{\eta}_{\Lambda,-}$ and $\breve{\eta}_{\Lambda,0}$ are independent, and the activity of the rods origins equals to the product of z on the probability to reach (in the case $\breve{\eta}_{\Lambda,0}$) or not to reach (in the case $\breve{\eta}_{\Lambda,-}$) the level $-T$. Distributions for the length of the rods are also properly changed . Moreover,

$$\breve{\eta}_{\Lambda,-} = \eta_{G^{-}_{\Lambda,T}},\ \eta_{G^{0}_{\Lambda,T}} = \breve{\eta}_{\Lambda,0} \cup \hat{\eta}_{\Lambda,0}.$$

Thus, configurations $\breve{\eta}_{\Lambda,-}$ form a field (M,ζ,p) defined by (43) and the union of configurations $\breve{\eta}_{\Lambda,0} \cup \hat{\eta}_{\Lambda,0}$ form again the field (M,ζ,p) defined by (42). $\square$

Formula (26) implies that only the distribution $\mathcal{P}^{0}_{G^{0}_{\Lambda,T},z}$ of the component $\eta_{G^{0}_{\Lambda,T}}$ is subjected to a reconstruction:

$$\mathcal{P}_{\Lambda,T,z} = \mathcal{P}_{G^{0}_{\Lambda,T},z} \times \mathcal{P}^{0}_{G^{+}_{\Lambda,T},z} \times \mathcal{P}^{0}_{G^{-}_{\Lambda,T},z} \times \mathcal{P}^{0}_{G_{\Lambda',\infty},z}, \tag{45}$$

where $\mathcal{P}_{G^{0}_{\Lambda,T},z}$ is assigned by a probability density analogous to (26):

$$\frac{d\mathcal{P}_{G^{0}_{\Lambda,T},z}}{d\mathcal{P}^{0}_{G^{0}_{\Lambda,T},z}}(\eta_{G^{0}_{\Lambda,T}}) = \frac{1}{Z_{\Lambda,T}} \exp\left\{ -\alpha \sum_{\{\xi_{j_1},\ \xi_{j_2}\} \subset \eta_{G^{0}_{\Lambda,T}}} \Phi^{T}(\xi_{j_1},\ \xi_{j_2}) \right\}. \tag{46}$$

We denote by

$$\Upsilon^{0} = \bigcup_{n=0}^{\infty} \Upsilon^{0}_{n} \tag{47}$$

a space of finite configurations of rods in R^{d+1}. Here Υ^0_n is a set of all n rods configurations, and $\Upsilon^0_0 = \{\emptyset\}$. Then the Lebesgue-Poisson measure $\lambda_{\zeta,p}$ wtih intensity $\zeta = \zeta(x)$ and distribution function $p = \{p_x(l)\}$ for the length of rods can be considered on the space Υ^0. This measure on each $\Upsilon^0_n,\ n = 0, 1, 2, \ldots$ is defined as follows

$$\int_{\Upsilon^0_n} f(\eta) d\lambda_{\zeta,p} = \frac{1}{n!} \int_{K^n} \tilde{f}(\xi_1, \ldots, \xi_n) \prod_{i=1}^{n} (-dp_{x_i}(l_i)) \prod_{i=1}^{n} \zeta(x_i)\ dx_i, \tag{48}$$

where $\xi_i = (x_i, l_i)$ are rods, $f(\eta)$ is a bounded function on Υ^0 with a finite support and $\tilde{f}(\xi_1, \ldots, \xi_n)$ is a symmetrical extension of $f(\eta)$ to the space K^n of ordered sequences of rods $(\xi_1, \ldots, \xi_n),\ \xi_i \cap \xi_j = \emptyset,\ i \neq j$. We will use the notation $\lambda_{\zeta,p} = \lambda_z$ in the case when $\zeta \equiv z,\ p_x(l) = e^{-l},\ l > 0, \forall x$. For any $G \subset K$ let $\Upsilon^0(G) \subset \Upsilon^0$ be a set of all finite configurations of rods from G, and $\lambda^G_{\zeta,p}$ be a restriction of the measure $\lambda_{\zeta,p}$ to the set $\Upsilon^0(G)$. Then for any two non-intersecting domains $G_1, G_2 \subset K$ we have

$$\Upsilon^0(G_1 \cup G_2) = \Upsilon^0(G_1) \times \Upsilon^0(G_2), \quad \lambda^{G_1 \cup G_2}_{\zeta,p} = \lambda^{G_1}_{\zeta,p} \times \lambda^{G_2}_{\zeta,p}. \tag{49}$$

Moreover, for any integrable functions $F(\eta),\ \phi_i(\eta),\ i = 1, \ldots, m$ defined on the space Υ^0 the following equality holds, see for example [12],

$$\int_{\Upsilon^0} \left(F(\eta) \sum\nolimits^{(\eta)}_{(\eta_1,\ldots,\eta_m)} \phi_1(\eta_1) \ldots \phi_m(\eta_m) \right) d\lambda_{\zeta,p}(\eta) \ = \tag{50}$$

$$\underbrace{\int_{\Upsilon^0} \cdots \int_{\Upsilon^0}}_{m \text{ times}} F\left(\eta_1 \cup \ldots \cup \eta_m\right) \prod_{i=1}^{m} \phi_i(\eta_i) \prod_{i=1}^{m}\ d\lambda_{\zeta,p}(\eta_i).$$

Here the sum in the left-hand side of (50) is taken over all ordered sets $(\eta_1, \ldots, \eta_m)$ of m nonempty finite configurations $\eta_i \in \Upsilon^0$ such that $\eta_i \cap \eta_j = \emptyset,\ i \neq j$, and $\cup^m_1 \eta_i = \eta$.

In what follows we will take $G \subset G^0_{\Lambda,T}$ and will write for simplicity $\lambda^G = \lambda^G_{\zeta,p}$, where $\zeta(x),\ p = \{p_x(l)\}$ are defined by formulas (42). The measure $\lambda^G(\Upsilon^0(G))$ for any bounded domain $G \subset G^0_{\Lambda,T}$ is equal to

$$\lambda^G(\Upsilon^0(G)) \ = \ \exp\left\{ \int_G \zeta(x)\ (-dp_x(l))\ dx \right\}. \tag{51}$$

In the case $G = G^0_{\Lambda,T}$ $(= G^{int}_M,\ M = \Lambda \times (-T,T))$

$$\int\limits_{G^0_{\Lambda,T}} \zeta(x)\ (-dp_x(l))\ dx \ = \ (2T+1)|\Lambda| z, \tag{52}$$

where $|\Lambda|$ is a d-dimensional volume of the domain $\Lambda \subset R^d$.

In the case $G = G^{loc}_{M=\Lambda\times(-T,T)}$

$$\int\limits_{G^{loc}_M} \zeta(x)\ (-dp_x(l))\ dx \ = \ 2T|\Lambda| z.$$

Lemma 4. *The probability measure $\mathcal{P}^0_{G^0_{\Lambda,T},z}$ on $\Upsilon'_{G^0_{\Lambda,T}}$ defined by (42) is equal to*

$$\mathcal{P}^0_{G^0_{\Lambda,T},z} = e^{-(2T+1)|\Lambda| z}\ \lambda^{G^0_{\Lambda,T}}. \tag{53}$$

For the proof see Appendix.

Next we consider the probability density

$$\hat{p}_{\Lambda,T,z}(\eta) = \frac{d\mathcal{P}_{G^0_{\Lambda,T},z}}{d\lambda^{G^0_{\Lambda,T}}}(\eta) \tag{54}$$

for $\mathcal{P}_{G^0_{\Lambda,T},z}$ with respect to the measure $\lambda^{G^0_{\Lambda,T}}$ (instead of the measure $\mathcal{P}^0_{G^0_{\Lambda,T},z}$). Density (54) can be defined again by formula (46) where the new normalizing factor $\hat{Z}_{\Lambda,T}$ is related with the normalizing factor from (46) by the following way

$$\hat{Z}_{\Lambda,T} = Z_{\Lambda,T}\ e^{(2T+1)|\Lambda| z}.$$

Let us note that for any bounded $G \subset K$ the sets Υ'_G and $\Upsilon^0(G)$ are the same up to a set with zero λ^G measure, consequently the distributions $\mathcal{P}^0_{G^0_{\Lambda,T},z}$ and $\mathcal{P}_{G^0_{\Lambda,T},z}$ could be considered as distributions on $\Upsilon^0(G^0_{\Lambda,T})$.

4 Related cluster expansion

At the beginning of the section we formulate conditions on the potential $\varphi(u)$

$$\text{Positivity:} \quad \varphi(u) \geq 0; \tag{55}$$

boundedness and fast decreasing on the infinity:

$$a)\ \varphi(u) < \frac{c}{(1+|u|)^{2m}}, \quad m > d, \quad \text{or} \quad b)\ \varphi(u) < c\, e^{-k|u|} \tag{56}$$

with an absolute constant $c > 0$. The last inequality (56) can be rewritten as

$$\varphi(u) < C \exp\{-2\hat{\varrho}(0,u)\}, \tag{57}$$

where

$$\hat{\varrho}(s_1, s_2) = \begin{cases} m \ln(1 + |s_1 - s_2|), & \text{in case a,} \\ \frac{k}{2}|s_1 - s_2|, & \text{in case b} \end{cases} \tag{58}$$

is a metrics in R^d, $s_1, s_2 \in R^d$ (see (34)).

Further we will follow constructions from the book [12]. The density (54) $\hat{p}_{\Lambda,T,z}(\eta)$, $\eta \in \Upsilon^0(G^0_{\Lambda,T}) \equiv \Upsilon_{\Lambda,T}$ has the following representation

$$\hat{p}_{\Lambda,T,z}(\eta) = \begin{cases} \hat{Z}^{-1}_{\Lambda,T}, & \eta = \emptyset, \\ \hat{Z}^{-1}_{\Lambda,T} \sum^{(\eta)}_{\{\eta_1,\ldots,\eta_m\}} \prod_{i=1}^m K^T(\eta_i), & \eta \neq \emptyset, \end{cases} \tag{59}$$

where the sum $\sum^{(\eta)}_{\{\eta_1,\ldots,\eta_m\}}$ is taken over all partitions of the finite configuration of rods η, i.e. over all unordered sets of mutually-disjoint configurations $\eta_1, \ldots, \eta_m$, $\eta_i \subseteq \eta$, $i = 1, \ldots, m$, $m = 1, 2, \ldots$, such that $\cup \eta_i = \eta$. We will use further the following designation: $\{\eta_1, \ldots, \eta_m\}$ for unordered sets and $(\eta_1, \ldots, \eta_m)$ for ordered sets of configurations.

The cluster weight $K^T(\eta)$ is equal to

$$K^T(\eta) = \begin{cases} 1, & |\eta| = 1, \\ \sum^{(\eta)}_{\sigma} \kappa^T_\sigma, & |\eta| \geq 2 \end{cases} \tag{60}$$

where $|\eta|$ is the number of rods in the configuration η, the sum $\sum^{(\eta)}_{\sigma}$ is taken over all connected graphs σ with the set of nodes $V(\sigma) = \{\xi_1, \ldots, \xi_s\} = \eta$, and

$$\kappa^T_\sigma = \prod_{\langle \xi_i, \xi_j \rangle \in \sigma} \left(e^{-\alpha\, \Phi^T(\xi_i,\xi_j)} - 1 \right), \tag{61}$$

where the product is over all edges $\langle \xi_i, \xi_j \rangle$ of the graph σ.

Lemma 5. *For any stable potential φ the cluster weight meets the following bound as $|\eta| \geq 2$*

$$|K^T(\eta)| \leq e^{\alpha B \sum\limits_{\xi \in \eta} l(\xi)} \sum\nolimits^{(\eta)}_{\mathcal{T}} \prod_{\langle \xi_i, \xi_j \rangle \in \mathcal{T}} \left(1 - e^{-\alpha\, |\Phi^T(\xi_i,\xi_j)|} \right), \tag{62}$$

where the sum is taken over all trees with the vertex set $V(\mathcal{T}) = \eta$, $l(\xi)$ is the length of the rod ξ, B is a constant.

Under $\varphi \geq 0$ (B =0) bound (62) can be rewritten as

$$|K^T(\eta)| \;\leq\; \sum\nolimits_{\mathcal{T}}^{(\eta)} \prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}} \left|\left(e^{-\alpha\,\Phi^T(\xi_i,\xi_j)} - 1\right)\right|. \tag{63}$$

where the sum is taken over all trees with the same vertex set $V(\mathcal{T}) = \eta$ as above.

For the proof see attachment.

We formulated Lemma 5 for the general case of a stable potential because bound (62) is the crucial point in the proof of Theorem 4. Further line of the proof is well adapted to the general case, and we return here again to the case of a non-negative potential $\varphi \geq 0$ to make our reasoning more simple.

Then

$$\hat{Z}_{\Lambda,T} = 1 + \int_{\Upsilon^0(G^0_{\Lambda,T})\setminus\emptyset} \sum\nolimits_{\{\eta_1,\ldots,\eta_m\}}^{(\eta)} \prod_{i=1}^{m} K^T(\eta_i) d\lambda^{G^0_{\Lambda,T}}(\eta). \tag{64}$$

Using equality (50), we have

$$\hat{Z}_{\Lambda,T} \;=\; 1 \;+\; \sum_{m=1}^{\infty} \frac{1}{m!} \left(\int_{\Upsilon^0(G^0_{\Lambda,T})\setminus\emptyset} K^T(\eta)\; d\lambda^{G^0_{\Lambda,T}}(\eta) \right)^m \;= \tag{65}$$

$$\exp\left\{ \int_{\Upsilon^0(G^0_{\Lambda,T})\setminus\emptyset} K^T(\eta)\; d\lambda^{G^0_{\Lambda,T}}(\eta) \right\}.$$

For any bounded set $G \subset G^0_{\Lambda,T}$ the space $\Upsilon^0(G^0_{\Lambda,T})$ can be written as

$$\Upsilon^0(G^0_{\Lambda,T}) = \Upsilon^0(G) \times \Upsilon^0(G^0_{\Lambda,T} \setminus G). \tag{66}$$

We denote a distribution on $\Upsilon^0(G)$ generated by the distribution $\mathcal{P}_{G^0_{\Lambda,T},z}$ (46) as $\mathcal{P}^G_{\Lambda,T,z}$.

Lemma 6. *The probability density $p^G_{\Lambda,T,z}(\eta) = \frac{d\mathcal{P}^G_{\Lambda,T,z}}{d\lambda^G}$, $\eta \in \Upsilon^0(G)$ is equal to*

$$p^G_{\Lambda,T,z}(\eta) \;=\; f^G_{\Lambda,T} \begin{cases} 1, & \eta = \emptyset, \\ \sum\nolimits^{(\eta)}_{\{\eta_1,\ldots,\eta_m\}} \prod_{i=1}^m r^G_{\Lambda,T}(\eta_i), & \eta \neq \emptyset, \end{cases} \tag{67}$$

where the sum is taken over partitions of the configuration η, and

$$r^G_{\Lambda,T}(\eta) \;=\; \int\limits_{\Upsilon^0(G^0_{\Lambda,T}\setminus G)} K^T(\eta\cup\bar\eta)\; d\lambda^{G^0_{\Lambda,T}\setminus G}(\bar\eta), \tag{68}$$

$$f^G_{\Lambda,T} = \exp\left\{-\int\limits_{\Upsilon^0(G)\setminus\emptyset} r^G_{\Lambda,T}(\eta)\; d\lambda^G(\eta)\right\} = \tag{69}$$

$$\exp\left\{-\int\limits_{\Upsilon^0(G)\setminus\emptyset} d\lambda^G(\eta) \int\limits_{\Upsilon^0(G^0_{\Lambda,T}\setminus G)} K^T(\eta\cup\bar\eta)\; d\lambda^{G^0_{\Lambda,T}\setminus G}(\bar\eta)\right\}.$$

Proof of Lemma 6 follows the same line as in [12].

It follows from (67) that the average of any bounded function F on $\Upsilon^0(G)$ with a bounded set $G\subset G^0_{\Lambda,T}$ equals to

$$\langle F\rangle_{\mathcal{P}_{\Lambda,T,z}} \;=\; \langle F\rangle_{\mathcal{P}^G_{\Lambda,T,z}} \;= \tag{70}$$

$$f^G_{\Lambda,T}\left[F(\emptyset) + \int\limits_{\Upsilon^0(G)\setminus\{\emptyset\}} \left(F(\eta)\sum\nolimits^{(\eta)}_{\{\eta_1,\dots,\eta_m\}} \prod_{i=1}^m r^G_{\Lambda,T}(\eta_i)\right) d\lambda^G(\eta)\right] =$$

$$f^G_{\Lambda,T}\left[F(\emptyset) + \sum_{m=1}^\infty \frac{1}{m!}\int\limits_{\Upsilon^0(G)\setminus\{\emptyset\}}\dots\int\limits_{\Upsilon^0(G)\setminus\{\emptyset\}} F\left(\bigcup_{i=1}^m \eta_i\right)\prod_{i=1}^m r^G_{\Lambda,T}(\eta_i)\prod_{i=1}^m d\lambda^G(\eta_i)\right] =$$

$$f^G_{\Lambda,T}\left[F(\emptyset) + \sum_{m=1}^\infty \frac{1}{m!}\int\limits_{(\Upsilon^0(G)\setminus\{\emptyset\})^m}\;\int\limits_{(\Upsilon^0(G^0_{\Lambda,T}\setminus G))^m} F\left(\bigcup_{i=1}^m \eta_i\right)\prod_{i=1}^m K^T(\eta_i\cup\bar\eta_i)\right.$$

$$\left.\prod_{i=1}^m d\lambda^G(\eta_i)\;\prod_{i=1}^m d\lambda^{G^0_{\Lambda,T}\setminus G}(\bar\eta_i)\right].$$

For any bounded set $\Lambda\subset R^d$ we introduce a "tube"

$$G_{\Lambda,\infty} = \Lambda\times\{-\infty,\infty\}\times R^1_+ \subset K$$

in the space of rods.

Lemma 7. *There exist the following limits as $T \to \infty$*
1) for any bounded domain $G \subset G_{\Lambda,\infty}$:

$$\lambda^G_{\zeta,p} \to \lambda^G_z, \tag{71}$$

where $\lambda^G_{\zeta,p}$ is the Lebesque-Poisson measure defined by (42), and λ^G_z is the Lebesque-Poisson measure with parameters $\zeta(x) \equiv z$ and $p_x(l) = e^{-l}$, see (41);
2) for any finite configuration $\eta \in \Upsilon^0$

$$K^T(\eta) \to K(\eta), \tag{72}$$

where $K(\eta)$ is defined similarly to $K^T(\eta)$, see (60) - (61) with $\Phi(\xi_1, \xi_2)$ instead of the function $\Phi^T(\xi_1, \xi_2)$, see (27);
3) for any bounded domain $G \subset G_{\Lambda,\infty}$:

$$f^G_{\Lambda,\infty} = \lim_{T\to\infty} f^G_{\Lambda,T} =$$

$$\exp\left\{ - \int\limits_{\Upsilon^0(G)\setminus\{\emptyset\}} \int\limits_{\Upsilon^0(G_{\Lambda,\infty}\setminus G)} K(\eta \cup \bar\eta)\ d\lambda^G_z(\eta)\ d\lambda^{G_{\Lambda,\infty}\setminus G}(\bar\eta) \right\} \tag{73}$$

where $f^G_{\Lambda,T}$ is defined in (69);
4)

$$\lim_{T\to\infty} \langle F \rangle_{\mathcal{P}^G_{\Lambda,T,z}} = \langle F \rangle_{\mathcal{P}_{\Lambda,\infty,z}} \tag{74}$$

for any bounded domain $G \subset G_{\Lambda,\infty}$ and a bounded local function F. Here

$$\langle F \rangle_{\mathcal{P}_{\Lambda,\infty,z}} = f^G_{\Lambda,\infty} \left(F(\emptyset) + \sum_{m=1}^{\infty} \frac{1}{m!} \int\limits_{(\Upsilon^0(G)\setminus\{\emptyset\})^m} \prod_{i=1}^m d\lambda^G(\eta_i) \right.$$

$$\left. \int\limits_{(\Upsilon^0(G_{\Lambda,\infty}\setminus G))^m} \prod_{i=1}^m d\lambda^{G_{\Lambda,\infty}\setminus G}(\bar\eta_i) F\left(\bigcup_{i=1}^m \eta_i\right) \prod_{i=1}^m K(\eta_i \cup \bar\eta_i) \right).$$

Proof. The statements of items 1) and 2) are clear.
3) The integral in (69) can be bounded from above by

$$\left| \int\limits_{\Upsilon^0(G)\setminus\{\emptyset\}} d\lambda^G_z(\eta) \int\limits_{\Upsilon^0(G^0_{\Lambda,T}\setminus G)} K^T(\eta \cup \bar\eta)\ d\lambda_z^{G^0_{\Lambda,T}\setminus G}(\bar\eta) \right| <$$

$$
\int\limits_{\Upsilon^0(G)\setminus\{\emptyset\}} d\lambda_z^G(\eta) \int\limits_{\Upsilon^0(G_{\Lambda,\infty}\setminus G)} d\lambda_z^{G_{\Lambda,\infty}\setminus G}(\bar\eta) \left(\sum_{\xi\in\eta}\chi_G(\xi)\right) |K^T(\eta\cup\bar\eta)| \ =
$$

$$
z\int\limits_G d\xi \int\limits_{\Upsilon^0(G)} d\lambda_z^G(\eta) \int\limits_{\Upsilon^0(G_{\Lambda,\infty}\setminus G)} d\lambda_z^{G_{\Lambda,\infty}\setminus G}(\bar\eta)\ |K^T(\xi\cup\eta\cup\bar\eta)| \ =
$$

$$
z\int\limits_G d\xi \int\limits_{\Upsilon^0(G_{\Lambda,\infty})} d\lambda_z^{G_{\Lambda,\infty}}(\eta)\ |K^T(\xi\cup\eta)|. \tag{75}
$$

Here $\chi_G(\xi),\ \xi=(x,l)$ is the characteristic function of $G,\ d\xi = dx(-dp(l)) = dx\ e^{-l}dl$ and we used in (75) general formula (49) and equality (50).

Next using estimate

$$
\left|e^{-\alpha\Phi^T(\xi_i,\xi_j)}-1\right| \ \le\ \left|e^{-\alpha\Phi(\xi_i,\xi_j)}-1\right|
$$

and estimate (63) we can continue

$$
z\int\limits_G d\xi \int\limits_{\Upsilon^0(G_{\Lambda,\infty})} d\lambda_z^{G_{\Lambda,\infty}}(\eta)\ |K^T(\xi\cup\eta)| \ =
$$

$$
z\ |G| \ +\ z\int\limits_G d\xi_0 \sum_{n=2}^\infty \frac{z^{n-1}}{(n-1)!} \int\limits_{K^{n-1}} |K^T(\xi_0,\xi_1,\ldots,\xi_{n-1})| d\xi_1\ldots d\xi_{n-1} \ <
$$

$$
z|G| + \sum_{n=2}^\infty \frac{z^n}{(n-1)!} \sum_{\substack{\mathcal{T}:V(\mathcal{T})=\\ \{\xi_0,\xi_1,\ldots,\xi_{n-1}\}}} \int\limits_G d\xi_0 \int\limits_{K^{n-1}} d\xi_1\ldots d\xi_{n-1} \prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}} \left|e^{-\alpha\Phi^T(\xi_i,\xi_j)}-1\right| \le
$$

$$
z|G| + \sum_{n=2}^\infty \frac{z^n}{(n-1)!} \sum_{\substack{\mathcal{T}:V(\mathcal{T})=\\ \{\xi_0,\xi_1,\ldots,\xi_{n-1}\}}} \int\limits_G d\xi_0 \int\limits_{K^{n-1}} d\xi_1\ldots d\xi_{n-1} \prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}} \left|e^{-\alpha\Phi(\xi_i,\xi_j)}-1\right| \le
$$

$$
z\ |G| \ +\ \sum_{n=2}^\infty \frac{z^n}{(n-1)!} \sum_{\substack{\kappa_0,\ldots,\kappa_{n-1}:\\ \kappa_0+\ldots+\kappa_{n-1}=2(n-1)}} \sum_{\substack{\mathcal{T}:V(\mathcal{T})=\{\xi_0,\ldots,\xi_{n-1}\}\\ r(i)=\kappa_i,\ i=0,\ldots,n-1}} \tag{76}
$$

$$
\int\limits_G d\xi_0 \int\limits_{K^{n-1}} \prod_{i=1}^{n-1} d\xi_i \prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}} \left|e^{-\alpha\Phi(\xi_i,\xi_j)}-1\right|.
$$

Here the summation in $\sum\limits_{\substack{\mathcal{T}:V(\mathcal{T})=\{\xi_0,\dots,\xi_{n-1}\}\\ r(i)=\kappa_i,\ i=0,\dots,n-1}}$ is over trees $\mathcal{T}$ with a set of points of the tree $\{\xi_0,\dots,\xi_{n-1}\}$ and vertex degrees $r(i)=\kappa_i>0,\ i=0,\dots,n-1$. We notice that any tree with n points has exactly $n-1$ edges and the sum of the vertex degrees equals to $2(n-1)$.

The integration over variables $\xi_0,\xi_1,\dots,\xi_{n-1}$ will operate recurrently. On the first step we will integrate over all "end variables", i.e. the variables ξ_i associated with the end points of the tree $\mathcal{T}^0=\mathcal{T}$ exceptint the variable ξ_0. Then we pass on to a new tree $\mathcal{T}^{(1)}$, which is constructed as a result of eliminating of all integrated on the first step points together with corresponding edges. If we continue this procedure we will integrate over all variables $\xi_1,\dots,\xi_{n-1}$ step by step, and ξ_0 will be the last variable for integration.

Let us estimate a result of the first integration over the "end variables" for the following integral (under given tree $\mathcal{T}^0$):

$$I(\mathcal{T}^0) \;=\; \int\limits_G d\xi_0 \int\limits_{K^{n-1}} \prod_{i=1}^{n-1} d\xi_i \prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}^0} \left|e^{-\alpha\Phi(\xi_i,\xi_j)}-1\right|. \tag{77}$$

We will show next that for a fixed rod $\xi\in K$ the integral

$$J(\xi) \;=\; \int\limits_K \left|e^{-\alpha\Phi(\xi,\bar\xi)}-1\right|\; d\bar\xi. \tag{78}$$

meets the following estimate

$$J(\xi) \;<\; 4\;\alpha\; R\; l(\xi) \tag{79}$$

with a length $l(\xi)$ of the rod ξ, and $R=\int_{R^d}\varphi(u)du$. We remind that $\Phi(\xi,\bar\xi)\ge 0,\ \alpha>0$, then putting $\bar\xi=\{(\bar s,\bar t),\bar l\}$ we have

$$J(\xi) \;<\; \alpha\int_K \varphi(s-\bar s)\;\Delta(\xi,\bar\xi)\; d\bar s\; d\bar t\; e^{-\bar l}\; d\bar l \;<\; \alpha\; R\int_{R^1\times R^1_+} \Delta(\xi,\bar\xi)\; d\bar t\; e^{-\bar l}\; d\bar l, \tag{80}$$

since $\Delta(\xi,\bar\xi)$ doesn't depend on s and $\bar s$. We consider now 4 cases to get an estimate on the integral in (80).

1. A projection of the rod $\bar\xi$ to the time axis t is entirely covered by a projection of the rod ξ, i.e. in the notation $\xi=\{(s,t),l\}$ we have $t<\bar t<$

$\bar t+\bar l<t+l$. In this case

$$\int_{R^1\times R^1_+}\Delta(\xi,\bar\xi)\ d\bar t\ e^{-\bar l}\ d\bar l=\int_0^l \bar l e^{-\bar l}d\bar l\int_0^{l-\bar l}d\bar t= \tag{81}$$

$$\int_0^l \bar l(l-\bar l)e^{-\bar l}d\bar l<l\int_0^\infty \bar l e^{-\bar l}d\bar l=l.$$

2. In the projection to the axis t the rods ξ and $\bar\xi$ are overlapping in such way that $t<\bar t<t+l<\bar t+\bar l$. Then

$$\int\Delta(\xi,\bar\xi)\ d\bar t\ e^{-\bar l}\ d\bar l=\int_0^l ue^{-u}du\int_0^\infty e^{-m}dm<l. \tag{82}$$

We used here new variables

$$\bar l=u+m,\ u=\Delta(\xi,\bar\xi),\ \bar t=l-u.$$

3. The similar case: the projections of rods ξ and $\bar\xi$ are overlapping in such way that $\bar t<t<\bar t+\bar l<t+l$. Then we have the same estimate as (82).

4. A projection of the rod ξ to the time axis t is entirely covered by a projection of the rod $\bar\xi$, i.e. $\bar t<t<t+l<\bar t+\bar l$. In this case

$$\int\Delta(\xi,\bar\xi)\ d\bar t\ e^{-\bar l}\ d\bar l<l\ e^{-l}\int_0^\infty e^{-m_1}dm_1\int_0^\infty e^{-m_2}dm_2<l. \tag{83}$$

Thus, (80) - (83) immediately imply (79). For any node i of the reminder tree, i.e. for nodes of a new tree $\mathcal{T}^1$, we denote by $K^{(1)}(i)$ the number of bonds of the tree $\mathcal{T}^0$ incident to the node i and eliminated on the first step. The above estimates imply the following bound on the integral $I(\mathcal{T}^0)$

$$I(\mathcal{T}^0)\ <\ \int\limits_G d\xi_0\int\limits_{K^{n_1-1}}\prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}^{(1)}}\left|e^{-\alpha\Phi(\xi_i,\xi_j)}-1\right| \tag{84}$$

$$\prod_{i\in V(\mathcal{T}^{(1)})}(4R\alpha l_i)^{K^{(1)}(i)}\prod_{\substack{i\in V(\mathcal{T}^{(1)})\\ i\neq 0}}d\xi_i,$$

where n_1 is a number of nodes of the tree $\mathcal{T}^{(1)}$. We notice that for end nodes $i\in V(\mathcal{T}^{(1)})$ of the tree $\mathcal{T}^{(1)}$, i.e. nodes eliminated on the second step of our procedure, we have

$$K^{(1)}(i)=\kappa_i-1,$$

where κ_i is the degree of the node i in the original tree $\mathcal{T}^{(0)}$. Thus, to continue the estimation of integral $I(\mathcal{T}^0)$ after the second step we have to find a bound for the following integral

$$J_\kappa(\xi) = \int \left| e^{-\alpha\Phi(\xi,\bar\xi)} - 1 \right| (4R\alpha\bar l)^{\kappa-1} d\bar\xi \ < \tag{85}$$

$$(4R\alpha)^{\kappa-1} R\alpha \int \Delta(\xi,\bar\xi)\ \bar l^{\kappa-1}\ e^{-\bar l}\ d\bar t\ d\bar l.$$

Let us consider again 4 cases as above.
1. In the first case

$$\int \Delta(\xi,\bar\xi)\bar l^{\kappa-1}\ e^{-\bar l}\ d\bar t\ d\bar l = \int_0^l \bar l^\kappa (l-\bar l) e^{-\bar l} d\bar l < l \int_0^\infty \bar l^\kappa e^{-\bar l} d\bar l = l\ \kappa! \tag{86}$$

2-3. Using the same change of variables as above

$$\bar l = u + m,\ u = \Delta(\xi,\bar\xi),\ \bar t = l - u,$$

we have

$$\int \Delta(\xi,\bar\xi)\ \bar l^{\kappa-1}\ e^{-\bar l}\ d\bar t\ d\bar l = \int_0^l u e^{-u} \int_0^\infty (m+u)^{\kappa-1} e^{-m}\ dm\ du < \tag{87}$$

$$l \int_0^\infty \int_0^\infty (m+u)^{\kappa-1} e^{-m-u}\ dm\ du = l \int_0^\infty v^\kappa e^{-v}\ dv = l\ \kappa!$$

4. Here

$$\int \Delta(\xi,\bar\xi)\ \bar l^{\kappa-1}\ e^{-\bar l}\ d\bar t\ d\bar l = l \int_0^\infty \int_0^\infty (m_1+m_2+l)^{\kappa-1} e^{-m_1-m_2-l}\ dm_1\ dm_2 < \tag{88}$$

$$l \int_0^\infty s^{\kappa-1} e^{-s}\ ds \int_0^{s-l} dm_1 < l \int_0^\infty s^\kappa e^{-s}\ ds = l\ \kappa!$$

where $s = m_1 + m_2 + l$.

Thus

$$J_\kappa(\xi) < (4R\alpha)^\kappa\ l\ \kappa!, \quad l = l(\xi), \tag{89}$$

and consequently,

$$I(\mathcal{T}^0)\ \le \int\limits_G d\xi_0 \int\limits_{K^{n_2-1}} \prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}^{(2)}} \left| e^{-\alpha\Phi(\xi_i,\xi_j)} - 1 \right| \tag{90}$$

$$\prod_{i\in V(\mathcal{T}^{(2)})} (4R\alpha l_i)^{K^{(2)}(i)} \prod_{j\in V(\mathcal{T}^{(1)}\setminus\mathcal{T}^{(2)})} \kappa_j!\ (4R\alpha)^{\kappa_j-1} \prod_{\substack{i\in V(\mathcal{T}^{(2)})\\ i\neq 0}} d\xi_i.$$

Here $K^{(2)}(i)$ is a number of bonds incident to the node $i \in V(\mathcal{T}^{(2)})$ and eliminated on the second step, $V(\mathcal{T}^{(1)})\setminus V(\mathcal{T}^{(2)})$ is a set of nodes of the tree $\mathcal{T}^{(1)}$ eliminated on the second step.

If we continue this procedure in the same way we get eventually the following estimate with the last node ξ_0

$$I(\mathcal{T}^0) \ \leq \ \int\limits_G l_0^\kappa \ d\xi_0 \ (4R\alpha)^{\kappa_0} \prod_{\substack{i\in V(\mathcal{T}^{(0)})\\ i\neq 0}} \left(\kappa_i!\ (4R\alpha)^{\kappa_i-1}\right) \ < \tag{91}$$

$$(4R\alpha)^{n-1} \int\limits_G l_0^\kappa \ d\xi_0 \prod_{\substack{i\in V(\mathcal{T}^{(0)})\\ i\neq 0}} \kappa_i!.$$

Here we used that $\sum_{i\in V(\mathcal{T}^{(0)})} \kappa_i = 2n-2$.

Let us estimate now

$$\int\limits_G l_0^\kappa \ d\xi_0.$$

We denote by $M \subset R^{d+1}$ a bounded closed subset of R^{d+1} such that all rods from G have some intersection with M. And let $G_M,\ G \subseteq G_M \subset K$ be a set of all rods from K intersecting M. We suppose that set M has no "time holes", i.e. each stright line parallel to the time axis intersects M at a point or in a segment.

Let $\Lambda \subset R^d$ be a projection of M to the space R^d. Then the following bound holds

$$\int\limits_G l_0^\kappa \ d\xi_0 \ \leq \int\limits_{G_M} l_0^\kappa \ d\xi_0 = (\kappa+1)!\ |\Lambda| \ + \ \kappa!\ |M|, \tag{92}$$

where $|\Lambda|$ is a d-dimensional volume of Λ, and $|M|$ is a $d+1$-dimensional volume of M. Indeed, we can represent G_M as a union of two nonoverlapping sets

$$G_M \ = \ G^0 \ \cup G^1,$$

where G^0 is a set of all rods with origins in M, and G^1 is a set of rods with origins outside of M (in the "past" of M). Then

$$\int\limits_{G^0} l_0^\kappa \, d\xi_0 \;\leq\; \int\limits_M dx \int\limits_0^\infty l^\kappa \, e^{-l} \, dl \;=\; \kappa! \, |M| \tag{93}$$

and

$$\int\limits_{G^1} l_0^\kappa \, d\xi_0 \;=\; |\Lambda| \int\limits_0^\infty \int\limits_0^\infty (r+s)^\kappa \, e^{-(r+s)} \, dr \, ds \;=\; (\kappa+1)! \, |\Lambda|. \tag{94}$$

Thus, estimate (91) together with (92) and bound $\kappa + 1 \leq n$ implies

$$I(\mathcal{T}^0) \;\leq\; \prod_{i \in V(\mathcal{T}^{(0)})} \kappa_i! \, (4R\alpha)^{n-1} (|M| + n|\Lambda|). \tag{95}$$

The number of all trees $\mathcal{T}$ with n vertecis and fixed vertex degrees $\{\kappa_i, \; i \in V(\mathcal{T})\}$ is equal, see [12, 15, 20],

$$\frac{(n-2)!}{\prod\limits_{i \in V(\mathcal{T})} (\kappa_i - 1)!} \;<\; \frac{2^n \, (n-2)!}{\prod\limits_{i \in V(\mathcal{T})} \kappa_i!}.$$

The number of ordered set $\{\kappa_i\}$ from n integer positive numbers such that

$$\kappa_1 + \ldots + \kappa_n = 2n - 2,$$

can be estimated from above by $C_{2n-3}^n \;<\; 2^{2n-3}$. Substituting these bounds to (76) we get for small enough α

$$z \int\limits_G d\xi \int\limits_{\Upsilon^0(G_{\Lambda,\infty})} d\lambda_z^{G_{\Lambda,\infty}}(\eta) \; |K(\xi \cup \eta)| \;\leq\; z\,|G| \; +$$

$$\sum_{n=2}^\infty \frac{z^n (n-2)!}{(n-1)!} (4R\alpha)^{n-1} 2^{3n-3} (|M| + n|\Lambda|) \;<\; (|M| + |\Lambda|) \, C(\alpha, z), \tag{96}$$

where $C(\alpha, z)$ is a constant doesn't depending on G. Finally, relations (72), (75), (96) and dominated convergence theorem imply (73).

Let us consider now the last statement of the Lemma. The expressions under the integral in the square brackets at two last lines in formula (70) converge as $T \to \infty$ to the expressions

$$F\left(\cup_{i=1}^{m} \eta_i\right) \prod_{i=1}^{m} K(\eta_i \cup \bar{\eta}_i).$$

Consequently, the integral in the square brackets in expression (70) is majorized by the following sum

$$\max_{\eta} |F(\eta)| \sum_{m=1}^{\infty} \frac{1}{m!} \int\limits_{(\Upsilon^0(G)\setminus\{\emptyset\})^m} \int\limits_{(\Upsilon^0(G^0_{\Lambda,\infty}\setminus G))^m} \prod_{i=1}^{m} K(\eta_i \cup \bar{\eta}_i) \tag{97}$$

$$\prod_{i=1}^{m} d\lambda^G(\eta_i) \prod_{i=1}^{m} d\lambda^{G^0_{\Lambda,\infty}\setminus G}(\bar{\eta}_i) \ <$$

$$\max_{\eta} |F(\eta)| \sum_{m=1}^{\infty} \frac{1}{m!} \left(\int\limits_{G} \int\limits_{\Upsilon^0(G_{\Lambda,\infty})} K(\xi \cup \eta)\ d\lambda^{G_{\Lambda,\infty}}(\eta)\ d\xi \right)^m \ <$$

$$\max_{\eta} |F(\eta)|\ \exp\{(|M| + |\Lambda|)\ C(\alpha, z)\}.$$

Thus Lemma 7 is proved completely.

Corollary. Formulae (67) - (69) imply that $\langle F \rangle$ is the average of the function F over a probability distribution $\mathcal{P}^G_{\Lambda,\infty,z}$ on the set $\Upsilon^0(G)$, where the probability density with respect to the measure λ^G_z is given by

$$p^G_{\Lambda,\infty,z}(\eta) = \begin{cases} f^G_{\Lambda,\infty}, & \eta = \emptyset, \\ f^G_{\Lambda,\infty} \ \sum^{(\eta)}_{\{\eta_1,\dots,\eta_s\}} \prod_i r^G_{\Lambda,\infty}(\eta_i), & \eta \neq \emptyset, \end{cases} \tag{98}$$

with

$$r^G_{\Lambda,\infty}(\eta) \ = \int\limits_{\Upsilon^0(G_{\Lambda,\infty}\setminus G)} K(\eta \cup \bar{\eta})\ d\lambda_z^{G_{\Lambda,\infty}\setminus G}(\bar{\eta}),$$

$$f^G_{\Lambda,\infty} \ = \ \exp\left\{ - \int\limits_{\Upsilon^0(G)} r^G_{\Lambda,\infty}(\eta)\ d\lambda^G_z(\eta) \right\}.$$

Let $G_0 \subset K$ is a set of rods intersecting hyperplane $Y_0 = \{(s,t) \in R^{d+1} : t = 0\}$. We can introduce in G_0 new coordinates $\xi = (s,\ l_-,\ l_+)$, where $s = \xi \cap Y_0 \in \Lambda \subset R^d$ is the point of intersection of the rod ξ with the hyperplane Y_0, $l_- = |t|,\ l_+ = l - |t|$ are lengths of two parts of the rod ξ lying to the left and to the right of the point s respectively. Then

$$d\xi \ = \ ds\ e^{-l_-} dl_-\ e^{-l_+} dl_+,$$

and the space $\Upsilon^0(G_0)$ can be considered as a space of finite configurations of pairs $\eta = \{(\xi_-,\ \xi_+)_i\}_i$ of rods with corresponding lengths $l_-,\ l_+$. These pairs are situated on the different sides of the hyperplane Y_0 and have the common end $s \in \Lambda$ lying on Y_0. The measure $\lambda_z^{G_0}$ on $\Upsilon^0(G_0)$ can be written as

$$d\lambda_z^{G_0}(\eta) \ = \ d\mu_z^\Lambda(\gamma) \prod_{s\in\gamma} e^{-l_-(s)} dl_-(s) \ \prod_{s\in\gamma} e^{-l_+(s)} dl_+(s), \quad \eta \in \Upsilon^0(G^0), \quad (99)$$

where $\gamma = \gamma(\eta) = \{s_i\}$ is a configuration of points of intersection of the rods $\xi \in \eta$ with the hyperplane Y_0, $d\mu_z^\Lambda(\gamma)$ is d-dimensional Lebesgue-Poisson measure with activity z on the set of finite configurations in Λ; $l_-(s),\ l_+(s)$ are lengths of the corresponding parts of the rod ξ "attached" at point $s \in \gamma$.

We denote by $\Pi_{\Lambda,\infty}^{G_0}$ a probability distribution on the set $\Gamma^0(\Lambda)$ of finite configurations inside Λ induced by the distribution $\mathcal{P}_{\Lambda,\infty}^{G_0}$ on $\Upsilon^0(G_0)$. From (99) it is seen that the density

$$\tilde{\omega}_{\Lambda,\infty,z}(\gamma) = \frac{d\Pi_{\Lambda,\infty}^{G_0}}{d\mu_z^\Lambda}$$

of this distribution with respect to d-dimensional Lebesgue-Poisson measure μ_z^Λ on $\Gamma^0(\Lambda)$ is equal to

$$\tilde{\omega}_{\Lambda,\infty,z}(\gamma) \ = \ \int p_{\Lambda,\infty}^{G_0}(\gamma, \{l_-(s), s \in \gamma\}, \{l_+(s), s \in \gamma\}) \qquad (100)$$

$$\prod_{s\in\gamma} e^{-l_-(s)} dl_-(s) \ \prod_{s\in\gamma} e^{-l_+(s)} dl_+(s),$$

where $p^{G_0}_{\Lambda,\infty}(\gamma, \{l_-(s), s \in \gamma\}, \{l_+(s), s \in \gamma\})$ is the same density $p^G_{\Lambda,\infty,z}(\eta)$ as in formula (98) rewritten in the new variables. Formulas (98) and (100) imply that

$$\tilde{\omega}_{\Lambda,\infty,z}(\gamma) = \varphi_\Lambda \times \begin{cases} 1, & \gamma = \emptyset, \\ \sum^{(\gamma)}_{\{\gamma_1,\ldots,\gamma_m\}} \prod_{i=1}^{m} \varrho_\Lambda(\gamma_i), & \gamma \neq \emptyset, \end{cases} \tag{101}$$

where $\sum^{(\gamma)}_{\{\gamma_1,\ldots,\gamma_m\}}$ is the same as above sum over all partitions of the configuration γ,

$$\varphi_\Lambda \;=\; \exp\left\{ - \int_{\Gamma^0(\Lambda)} \varrho_\Lambda(\gamma)\; d\mu^\Lambda_z(\gamma) \right\},$$

and $\varrho_\Lambda(\gamma)$ is defined as

$$\varrho_\Lambda(\gamma) = \int r^{G_0}_{\Lambda,\infty}(\gamma, \{l_-(s)\}, \{l_+(s)\}) \prod_{s\in\gamma} e^{-l_-(s)} dl_-(s) \prod_{s\in\gamma} e^{-l_+(s)} dl_+(s). \tag{102}$$

5 Representation for ground state Ψ_Λ

We find now another representation for the probability distribution $\Pi^{G_0}_{\Lambda,\infty}$ on the set $\Gamma^0(\Lambda)$. Let us consider the trajectory space $\hat{\Upsilon}(G_{\Lambda,T})$, then the density $\hat{\omega}^{G_0}_{\Lambda,T}$ of the distribution $\Pi^{G_0}_{\Lambda,T}$ on the space of configurations $\hat{\gamma} = \gamma_{t=0} \subset \Lambda$ with respect to the Poisson measure π_z on $\Gamma^0(\Lambda)$ can be written as

$$\hat{\omega}^{G_0}_{\Lambda,T}(\hat{\gamma}) = \frac{Z_{\Lambda,(-T,0)}(\hat{\gamma})\; Z_{\Lambda,(0,T)}(\hat{\gamma})}{Z_{\Lambda,(-T,T)}}, \tag{103}$$

where partition function $Z_{\Lambda,(-T,T)} = Z_{\Lambda,T}$ is defined by (25), and

$$Z_{\Lambda,(0,T)}(\hat{\gamma}) \;=\; \int_{\Upsilon^+_\Lambda(\hat{\gamma})} e^{-\alpha \int_0^T U_\Lambda(\gamma(t))dt} d\mathcal{P}^{0,+}_{\Lambda,z}(\{\gamma(t), t > 0\} | \hat{\gamma}), \tag{104}$$

$$Z_{\Lambda,(-T,0)}(\hat{\gamma}) \;=\; \int_{\Upsilon^-_\Lambda(\hat{\gamma})} e^{-\alpha \int_{-T}^0 U_\Lambda(\gamma(t))dt} d\mathcal{P}^{0,-}_{\Lambda,z}(\{\gamma(t), t < 0\} | \hat{\gamma}), \tag{105}$$

with $Z_{\Lambda,(-T,0)}(\hat{\gamma}) \;=\; Z_{\Lambda,(0,T)}(\hat{\gamma})$ and

$$Z_{\Lambda,(-T,T)} \;=\; \int\limits_{\Gamma^0(\Lambda)} Z_{\Lambda,(-T,0)}(\hat{\gamma})\; Z_{\Lambda,(0,T)}(\hat{\gamma})\; d\pi_z(\hat{\gamma}). \tag{106}$$

Here $\Upsilon^+_\Lambda(\hat{\gamma})$ is the space of trajectories $\{\gamma(t),\ t>0\}$ of the free Glauber dynamics considered in a "semi-tube" of future $\Lambda\times(0,\infty)$ with condition

$$\gamma(t=0)=\hat{\gamma}. \tag{107}$$

The space $\Upsilon^-_\Lambda(\hat{\gamma})$ is defined by the analogous way. Conditional distributions $\mathcal{P}^{0,\pm}_{\Lambda,z}(\cdot\mid\hat{\gamma})$ are generated by the distribution $\mathcal{P}^0_{\Lambda,z}$ on the space of trajectories of the free Glauber dynamics under condition (107), and the conditional distributions are given on the spaces $\Upsilon^\pm_\Lambda(\hat{\gamma})$ respectively; π_z is the stationary distribution on $\Gamma^0(\Lambda)$ of the free Glauber dynamics considered in the domain $\Lambda\times(-\infty,\infty)$, i.e. the Poisson measure on $\Gamma(\Lambda)$ with the intensity z.

As before we rewrite expressions (104) - (105) for partition functions using the ensemble of rods. Any configuration from $\Upsilon^+_{\Lambda,T}$ can be decomposed into a pair of configurations of rods $(\tilde{\eta}_{\hat{\gamma}},\eta)$, where $\tilde{\eta}_{\hat{\gamma}}=\{\xi_s,\ s\in\hat{\gamma}\}$ is a configuration of rods ξ_s attached to a corresponding point $s\in\hat{\gamma}$ of the point configuration $\hat{\gamma}\subset\Lambda$, and η is a configuration of "free" rods with origins from $\Lambda\times(0,T)\subset R^{d+1}$. Similarly, the space of configurations $\Upsilon^+_{\Lambda,\infty}=\Upsilon^+_\Lambda$ can be represented as a space of pairs of rods $(\tilde{\eta}_{\hat{\gamma}},\eta)$, where $\tilde{\eta}_{\hat{\gamma}}=\{\xi_s,\ s\in\hat{\gamma}\}$ is defined as above, and a "free" configuration η of rods with origins from $\Lambda\times(0,\infty)$ can be again decomposed into a pair of configurations with origins in $\Lambda\times(0,T)$ and in $\Lambda\times[T,\infty)$ respectively. Thus, the distribution $\mathcal{P}^{0,+}_{\Lambda,z}(\cdot\mid\hat{\gamma})$ on $\Upsilon^+_{\Lambda,\infty}$ is represented as a product

$$\mathcal{P}^{0,+}_{\Lambda,z}(\cdot\mid\hat{\gamma}) \;=\; \mathcal{P}^{0,+}_{\Lambda,T}(\cdot\mid\hat{\gamma})\times\mathcal{P}^{0,+}_{\Lambda,[T,\infty)}(\cdot\mid\hat{\gamma}),$$

and

$$d\mathcal{P}^{0,+}_{\Lambda,T}((\tilde{\eta}_{\hat{\gamma}},\eta)\mid\hat{\gamma}) \;=\; d\lambda_z^{G^+_{\Lambda,T}}(\eta)\prod_{s\in\hat{\gamma}} e^{-l(s)}dl(s)\; e^{-|\Lambda|Tz},$$

where $G^+_{\Lambda,T}$ is the set of rods with origins in $\Lambda\times(0,T)$. Then we get

$$Z_{\Lambda,(0,T)}(\hat{\gamma}) \;=\; \int\prod_{s\in\hat{\gamma}} e^{-l(s)}dl(s)\int d\lambda_z^{G^+_{\Lambda,T}}(\eta)\; e^{-\alpha\sum\limits_{\{\xi_i,\xi_j\}\subset\tilde{\eta}_{\hat{\gamma}}\cup\eta}\Phi^T(\xi_i,\xi_j)}\; e^{-|\Lambda|Tz}. \tag{108}$$

Using the same reasoning as above the exponent in (108) can be rewritten as

$$e^{-\alpha \sum\limits_{\{\xi_i,\xi_j\}\subset \tilde{\eta}_{\hat{\gamma}}\cup\eta} \Phi^T(\xi_i,\xi_j)} = \sum_m \sum\nolimits_{\{(\tilde{\eta}_1,\eta_1),\ldots,(\tilde{\eta}_m,\eta_m)\}}^{\tilde{\eta}_{\hat{\gamma}}\cup\eta} \prod_{i=1}^{m} K^T(\tilde{\eta}_i\cup\eta_i) = \tag{109}$$

$$\sum_m \sum_{r\le m} \sum\nolimits_{\{\tilde{\eta}_1,\ldots,\tilde{\eta}_r\}}^{\tilde{\eta}_{\hat{\gamma}}} \sum\nolimits_{(\eta_1,\eta_2)}^{\eta} \sum\nolimits_{(\eta_1^{(1)},\ldots,\eta_r^{(1)})}^{\eta_1} \prod_{i=1}^{r} K^T(\tilde{\eta}_i\cup\eta_i^{(1)})$$

$$\sum\nolimits_{\{\eta_1^{(2)},\ldots,\eta_{m-r}^{(2)}\}}^{\eta_2} \prod_{j=1}^{m-r} K^T(\eta_j^{(2)}).$$

Here the sum $\sum_{\{\tilde{\eta}_1,\ldots,\tilde{\eta}_r\}}^{\tilde{\eta}_{\hat{\gamma}}}$ is taken over all unordered partition of $\tilde{\eta}_{\hat{\gamma}}$, the sum $\sum_{(\eta_1,\eta_2)}^{\eta}$ is the sum over ordered decomposition of η into two configurations $\eta_1\cup\eta_2$, the sum $\sum_{(\eta_1^{(1)},\ldots,\eta_r^{(1)})}^{\eta_1}$ is taken over all ordered partitions of η_1 into r sub-configurations $\eta_i^{(1)}$ (which can be empty), and the last sum $\sum_{\{\eta_1^{(2)},\ldots,\eta_{m-r}^{(2)}\}}^{\eta_2}$ is defined by the similar way.

Using decomposition (109) we have

$$Z_{\Lambda\times[0,T]}(\hat{\gamma}) = \sum_m \sum\nolimits_{\{\hat{\gamma}_1,\ldots,\hat{\gamma}_m\}}^{(\hat{\gamma})} \prod_{i=1}^{m} \left(\int K^T(\tilde{\eta}_{\hat{\gamma}_i}\cup\eta)\, d\lambda_z^{G^+_{\Lambda,T}}(\eta) \prod_{s\in\hat{\gamma}_i} e^{-l_s}\, dl_s \right) \tag{110}$$

$$\exp\left\{ \int_{\Upsilon^0(\Lambda,T)} K^T(\eta)\, d\lambda_z^{G^+_{\Lambda,T}}(\eta) - z|\Lambda|T \right\} =$$

$$\sum_m \sum\nolimits_{\{\hat{\gamma}_1,\ldots,\hat{\gamma}_m\}}^{(\hat{\gamma})} \prod_{i=1}^{m} \hat{\varrho}_{\Lambda,T}(\hat{\gamma}_i)\, \exp\left\{ \int_{\Upsilon^0(\Lambda,T)} K^T(\eta)\, d\lambda_z^{G^+_{\Lambda,T}}(\eta) - z|\Lambda|T \right\},$$

where

$$\hat{\varrho}_{\Lambda,T}(\hat{\gamma}) = \int\limits_{\Upsilon^0(\Lambda,T)\times R_+^{|\hat{\gamma}|}} K^T(\tilde{\eta}_{\hat{\gamma}}\cup\eta)\, d\lambda_z^{G^+_{\Lambda,T}}(\eta) \prod_{s\in\hat{\gamma}} e^{-l_s}\, d\, l_s \tag{111}$$

with $\tilde{\eta}_{\hat{\gamma}} = \{(s,l_s),\ s\in\hat{\gamma}\}$. Then (106), (110) together with

$$d\pi_z(\hat{\gamma}) = d\mu_z^{\Lambda}(\hat{\gamma})\, e^{-z|\Lambda|}$$

imply that

$$Z_{\Lambda\times(-T,T)} \;=\; \exp\left\{2\int\limits_{\Upsilon^0(\Lambda,T)} K^T(\eta)\ d\lambda_z^{G^+_{\Lambda,T}}(\eta) - 2|\Lambda|Tz - |\Lambda|z\right\} \tag{112}$$

$$\int\left(\sum_m \sum\nolimits^{(\hat{\gamma})}_{\{\hat{\gamma}_1,\dots,\hat{\gamma}_m\}} \prod_{i=1}^m \hat{\varrho}_{\Lambda,T}(\hat{\gamma}_i)\right)^2\ d\mu_z^\Lambda(\hat{\gamma}).$$

Thus using (103) we get the following expression for the density $\hat{\omega}^{G_0}_{\Lambda,\infty}(\hat{\gamma})$

$$\hat{\omega}^{G_0}_{\Lambda,\infty}(\hat{\gamma}) \;=\; \lim_{T\to\infty}\frac{Z_{\Lambda\times(0,T)}(\hat{\gamma})\ Z_{\Lambda\times(-T,0)}(\hat{\gamma})}{Z_{\Lambda\times(-T,T)}} \;=\; \tag{113}$$

$$\frac{\left(\sum\limits_m \sum\nolimits^{(\hat{\gamma})}_{\{\hat{\gamma}_1,\dots,\hat{\gamma}_m\}} \prod\limits_{i=1}^m \hat{\varrho}_{\Lambda,\infty}(\hat{\gamma}_i)\right)^2\ e^{z|\Lambda|}}{\int\left(\sum\limits_m \sum\nolimits^{(\hat{\gamma})}_{\{\hat{\gamma}_1,\dots,\hat{\gamma}_m\}} \prod\limits_{i=1}^m \hat{\varrho}_{\Lambda,\infty}(\hat{\gamma}_i)\right)^2\ d\mu_z^\Lambda(\hat{\gamma})}$$

with

$$\hat{\varrho}_{\Lambda,\infty}(\hat{\gamma}) \;=\; \lim_{T\to\infty}\hat{\varrho}_{\Lambda,T}(\hat{\gamma}) = \int\limits_{\Upsilon^0(\Lambda,\infty)\times R_+^{|\hat{\gamma}|}} K(\tilde{\eta}_{\hat{\gamma}}\cup\eta)\ d\lambda_z^{G^+_{\Lambda,\infty}}(\eta)\prod_{s\in\hat{\gamma}} e^{-l_s}\ d\ l_s. \tag{114}$$

Thus, we get

$$d\Pi^{G_0}_{\Lambda,\infty}(\hat{\gamma}) \;=\; \hat{\omega}^{G_0}_{\Lambda,\infty}(\hat{\gamma})\ d\mu_z^\Lambda(\hat{\gamma})\ e^{-z|\Lambda|}.$$

On the other hand, as follows from the Feynman-Kac formula, we have for semigroup $\exp\{tH_\Lambda\}$

$$Z_{\Lambda\times(-T,T)} \;=\; \int\limits_{\Upsilon_{\Lambda,\infty}} \exp\left\{-\alpha\int\limits_{-T}^{T} U_\Lambda(\gamma(t))\ dt\right\}\ d\mathcal{P}_z^0(\sigma) \;=\; (e^{2TH_\Lambda}1,1)_{\mu_z^\Lambda}e^{-z|\Lambda|}, \tag{115}$$

where $(\cdot,\cdot)_{\mu_z^\Lambda}$ is the scalar product in $L_2(\Gamma^0(\Lambda),\mu_z^\Lambda)$. Similar to (115) we get

$$Z_{\Lambda\times(0,T)}(\hat{\gamma}) \;=\; Z_{\Lambda\times(-T,0)}(\hat{\gamma}) \;=\; (e^{TH_\Lambda}\ 1)(\hat{\gamma}). \tag{116}$$

Thus (113) implies that

$$\Psi_\Lambda(\hat\gamma) \;\equiv\; \lim_{T\to\infty} \frac{(e^{TH_\Lambda}\;1)(\hat\gamma)}{\|e^{TH_\Lambda}\;1\|_{L_2(\Gamma^0(\Lambda),\mu_z^\Lambda)}} \;=\; \lim_{T\to\infty} \frac{Z_{\Lambda\times(0,T)}(\hat\gamma)}{(Z_{\Lambda\times(-T,T)})^{1/2}} \tag{117}$$

exists and equals to

$$\Psi_\Lambda(\hat\gamma) \;=\; \frac{\sum\limits_m \sum_{\{\hat\gamma_1,\dots,\hat\gamma_m\}}^{(\hat\gamma)} \prod\limits_{i=1}^m \hat\varrho_{\Lambda,\infty}(\hat\gamma_i)\; e^{\frac{1}{2}z|\Lambda|}}{\left(\int\limits_{\Gamma^0(\Lambda)} \left(\sum\limits_m \sum_{\{\hat\gamma_1,\dots,\hat\gamma_m\}}^{(\hat\gamma)} \prod\limits_{i=1}^m \hat\varrho_{\Lambda,\infty}(\hat\gamma_i)\right)^2 \; d\mu_z^\Lambda(\hat\gamma)\right)^{1/2}}. \tag{118}$$

Consequently, (113) and (118) imply that $\hat\omega^{G_0}_{\Lambda,\infty}(\hat\gamma) = \Psi^2_\Lambda(\hat\gamma)$, and finally we have $\Pi^{G_0}_{\Lambda,\infty} = \nu_z^\Lambda$, where the measure ν_z^Λ was defined by formula (8).

6 Proof of Theorem 1

Using decomposition $L_2(\Gamma, d\pi_z) = L_2(\Gamma^0(\Lambda), d\pi_z^\Lambda) \otimes L_2(\Gamma^0(\Lambda'), d\pi_z^{\Lambda'})$ and equality $d\pi_z^\Lambda = d\mu_z^\Lambda \; e^{-z|\Lambda|}$ we can reduce the problem of self-adjointness for the operator H_Λ on $L_2(\Gamma, d\pi_z)$ to the same problem for the operator H'_Λ on $\mathcal{H}_\Lambda = L_2(\Gamma^0(\Lambda), \mu_z^\Lambda)$ acting as follows

$$(H'_\Lambda\;\Phi)_n(s_1,\dots,s_n) \;=\; \sum_{i=1}^n \Phi_{n-1}(s_1,\dots,\breve{s}_i\dots s_n) - \tag{119}$$

$$(\alpha\, U_\Lambda(s_1,\dots,s_n)+n+z|\Lambda|)\;\Phi_n(s_1,\dots,s_n)+z\int_\Lambda \Phi_{n+1}(s_1,\dots,s_n,s)\,ds,\;\; n\ge 1,$$

$$(H'_\Lambda\;\Phi)_0 \;=\; -z\Phi_0 + z\int_\Lambda \Phi_1(s)\;ds, \quad n=0.$$

Here $\breve{s}_i$ means that the variable s_i is omitted,

$$\Phi \;=\; (\Phi_0, \Phi_1(s_1), \dots\; \Phi_n(s_1,\dots,s_n), \dots) \in L_2(\Gamma^0(\Lambda)), \tag{120}$$

$\Phi_n(s_1,\dots,s_n)$ is the value of $\Phi(\gamma)$ on the stratum $\Gamma^0_n(\Lambda) \subset \Gamma^0(\Lambda)$ of n-points configurations $\gamma = (s_1,\dots,s_n)$, $s_i \ne s_j$, and $U_\Lambda(s_1,\dots,s_n) \ge 0$. The operator H'_Λ is a symmetrical operator on the set $D_{fin} \subset \mathcal{H}_\Lambda$ of finite vectors, i.e. vectors with $\Phi_n = 0$ for all $n > N = N(\Phi)$. Thus, H'_Λ is a closable operator. For the closure, we will use the same notation H'_Λ.

To complete the proof of the self-adjointness we should check that for large enough $\xi > 0$ the range $Ran\ (H'_\Lambda - \xi E) = \mathcal{H}_\Lambda$, or what is equivalent that the equation

$$(H'_\Lambda \ - \ \xi E)\ F \ = \ G \tag{121}$$

is solvable for large enough $\xi > 0$ and all $G \in \mathcal{H}_\Lambda$. We can rewrite H'_Λ as $H'_\Lambda \ = \ T \ + \ R$ separating the diagonal (over number of variables s_i) part T of H'_Λ. Then positivity of T implies that (121) is equivalent to the equation

$$(T - \xi E)(E + (T - \xi E)^{-1} R)\ F \ = \ G. \tag{122}$$

If

$$\|(T - \xi E)^{-1} R)\| \ < \ 1, \tag{123}$$

then the solution of equation (122) can be found as

$$F \ = \ (E + (T - \xi E)^{-1} R)^{-1}\ (T - \xi E)^{-1}\ G.$$

Thus, we should prove inequality (123).

Denote by $\mathcal{H}_{\Lambda,n} \subset \mathcal{H}_\Lambda$ a subspace of sequences (120) with $\Phi_k = 0,\ k \neq n$, then the norm in $\mathcal{H}_{\Lambda,n}$ is defined as

$$\|\Phi_n\|^2_{\mathcal{H}_{\Lambda,n}} \ = \ \frac{z^n}{n!} \int\limits_{\Lambda^n} \Phi_n^2(s_1, \ldots, s_n) ds_1 \ldots ds_n.$$

For any $n \geq 1$ we have

$$\left\| \frac{\sum_{i=1}^n \Phi_{n-1}(s_1, \ldots, \breve{s}_i \ldots s_n)}{(\alpha\ U_\Lambda(s_1, \ldots, s_n) + n + z|\Lambda| + \xi)} \right\|_{\mathcal{H}_{\Lambda,n}} \ < \ \frac{\sqrt{nz|\Lambda|}\ \|\Phi_{n-1}\|_{\mathcal{H}_{\Lambda,n-1}}}{n + z|\Lambda| + \xi}.$$

Consequently,

$$\left\| \frac{\sum_{i=1}^n \Phi_{n-1}(s_1, \ldots, \breve{s}_i \ldots s_n)}{(\alpha\ U_\Lambda(s_1, \ldots, s_n) + n + z|\Lambda| + \xi)} \right\|^2_{\mathcal{H}_{\Lambda,n}} \ < \ \frac{nz|\Lambda|}{(n + z|\Lambda| + \xi)^2}\ \|\Phi_{n-1}\|^2_{\mathcal{H}_{\Lambda,n-1}} \ <$$

$$\max_n \frac{nz|\Lambda|}{(n + z|\Lambda| + \xi)^2}\ \|\Phi_{n-1}\|^2_{\mathcal{H}_{\Lambda,n-1}}. \tag{124}$$

Similarly, we get

$$\left\| \frac{z \int\limits_{\Lambda} \Phi_{n+1}(s_1, \ldots, s_n, s) ds}{(\alpha\, U_\Lambda(s_1, \ldots, s_n) + n + z|\Lambda| + \xi)} \right\|^2_{\mathcal{H}_{\Lambda,n}} < \frac{(n+1)z|\Lambda|}{(n + z|\Lambda| + \xi)^2} \, \|\Phi_{n+1}\|^2_{\mathcal{H}_{\Lambda,n+1}} <$$

$$\max_n \frac{(n+1)z|\Lambda|}{(n + z|\Lambda| + \xi)^2} \, \|\Phi_{n+1}\|^2_{\mathcal{H}_{\Lambda,n+1}}. \tag{125}$$

Finally (119), (124), (125) imply that

$$\|(T - \xi E)^{-1} R\| \; < \; 2\sqrt{\max_n \frac{(n+1)z|\Lambda|}{(n + z|\Lambda| + \xi)^2}} \; < \; 1$$

for all large enough ξ. The first statement of Theorem 1 is proved.

We will use the following proposition in the proof of the second statement of the theorem.

Proposition [11]. *Let H be a bounded from above self-adjoint operator in a Hilbert space $L_2(\Omega, \mu)$, where (Ω, μ) is a space with a finite measure μ, the semigroup* $\exp\{tH\}$ *meets the condition of improving positivity (see [16]), and the limit*

$$\lim_{T \to \infty} \frac{e^{TH} \; 1}{\|e^{TH} \; 1\|} \; = \; \Psi \tag{126}$$

exists, such that $(\Psi, 1) > 0$. Then Ψ is a unique ground state of the operator H and $\Psi > 0$.

In our case we see from (117) - (118) that (126) holds and

$$(\Psi_\Lambda(\hat{\gamma}), \, 1)_{L_2(\Gamma^0(\Lambda), \mu_z^\Lambda)} \; = \; \frac{\exp\left\{ \int\limits_{\Gamma^0(\Lambda)} \hat{\varrho}_{\Lambda,\infty}(\hat{\gamma}) \; d\mu_z^\Lambda(\hat{\gamma}) + \frac{1}{2} z|\Lambda| \right\}}{\left(\int\limits_{\Gamma^0(\Lambda)} \left(\sum\limits_m \sum\nolimits^{(\hat{\gamma})}_{\{\hat{\gamma}_1, \ldots, \hat{\gamma}_m\}} \prod\limits_{i=1}^m \hat{\varrho}_{\Lambda,\infty}(\hat{\gamma}_i) \right)^2 \; d\mu_z^\Lambda(\hat{\gamma}) \right)^{1/2}} \; > \; 0$$

consequently, the operator H_Λ has a unique ground state $\Psi_\Lambda(\gamma) \; > \; 0$. Thus, the second state of Theorem 1 is proved.

7 Proof of Theorem 4

7.1 First statement

As follows from representation (98) probability measures $\{\mathcal{P}^G_{\Lambda,\infty,z},\ G \subset G_{\Lambda,\infty}\}$ constructed in Lemma 7 form an consistent family of measures, i.e. for any two bounded sets $G_1 \subset G_2 \subset G_{\Lambda,\infty}$ we have:

$$\mathcal{P}^{G_2}_{\Lambda,\infty,z}|_{\Upsilon^0(G_1)} \ = \ \mathcal{P}^{G_1}_{\Lambda,\infty,z}.$$

That implies, see [13], that a probability measure $\tilde{\mathcal{P}}_{\Lambda,\infty,z}$ exists on $\Upsilon(G_{\Lambda,\infty})$, such that for any local function F_G with $G \subseteq G^{int}_M \subset G_{\Lambda,\infty}$ (with some bounded set $M \subset \Lambda \times (-\infty, \infty)$) :

$$\lim_{T\to\infty} \langle F_G \rangle_{\mathcal{P}_{\Lambda,T,z}} \ = \ \langle F_G \rangle_{\mathcal{P}^G_{\Lambda,\infty,z}} \ = \ \langle F_G \rangle_{\tilde{\mathcal{P}}_{\Lambda,\infty,z}}.$$

If we consider a measure on the space Υ of the following form

$$\mathcal{P}_{\Lambda,\infty,z} \ = \ \tilde{\mathcal{P}}_{\Lambda,\infty,z} \times \mathcal{P}^0_{\Lambda',\infty,z} \tag{127}$$

where $\mathcal{P}^0_{\Lambda',\infty,z}$ is the distribution of the free dynamics on Λ', we get (30).

We will show next that a distribution $\hat{\mathcal{P}}_{\Lambda,\infty,z}$ of the process associated with the semigroup $\tilde{S}^\Lambda_t = \exp\{t\tilde{H}_\Lambda\}$ (i.e. the Markov process $\mathcal{G}_\Lambda$ from Theorem 2) is the same as the distribution $\mathcal{P}_{\Lambda,\infty,z}$ (127).

Lemma 8. *The following asymptotics hold as $T \to \infty$*

$$Z_{\Lambda,T}(\gamma) \ = \ e^{\lambda^0_\Lambda T}\Psi_\Lambda(\gamma)(\Psi_\Lambda(\gamma), 1)_{\pi_z} \ + \ \delta) \tag{128}$$

with $\|\delta\|_{L_2(\Gamma^0(\Lambda),\pi_z)} \ = \ o(1)$,

$$Z_{\Lambda\times(-T,T)} \ = \ e^{2\lambda^0_\Lambda T}\left((\Psi_\Lambda(\gamma), 1)^2_{\pi_z} \ + \ o(1)\right), \tag{129}$$

where λ^0_Λ is the eigenvalue corresponding to the ground state Ψ_Λ, see Theorem 1.

Proof. Let $\mathcal{H}^\perp \subset L_2(\Gamma^0(\Lambda), \pi_z)$ be the orthogonal complement to the vector Ψ_Λ, and $H^\perp_\Lambda$ be a restriction of the operator H_Λ to the space $\mathcal{H}^\perp$. Then formula (115) implies

$$(e^{TH_\Lambda}\ 1)(\gamma) \ = \ e^{\lambda^0_\Lambda T}\ \Psi_\Lambda(\gamma)\ (\Psi_\Lambda(\gamma), 1)_{\pi_z} \ + \ e^{H^\perp_\Lambda T} 1^\perp(\gamma),$$

where $1^\perp$ is the projection of the vector 1 to the space $\mathcal{H}^\perp$. We estimate now the norm of the second term:

$$\|e^{H_\Lambda^\perp T}1^\perp\|^2 \;=\; \int\limits_{-\infty}^{\lambda_\Lambda^0} e^{2\lambda T} d\sigma_{1^\perp}(\lambda) \;=$$

$$e^{2\lambda_\Lambda^0 T}\left(\int\limits_{-\infty}^{\lambda_\Lambda^0 - a} e^{2(\lambda-\lambda_\Lambda^0)T} d\sigma_{1^\perp}(\lambda) \;+\; \int\limits_{\lambda_\Lambda^0 - a}^{\lambda_\Lambda^0} e^{2(\lambda-\lambda_\Lambda^0)T} d\sigma_{1^\perp}(\lambda)\right). \qquad (130)$$

Here $\sigma_{1^\perp}(\Delta)$ is the spectral measure of the operator $H_\Lambda^\perp$ on the vector $1^\perp$. Then the first term in the bracket in (130) is less then e^{-2aT} and the second term is estimated from above by $\sigma_{1^\perp}(\lambda_\Lambda^0 - a, \lambda_\Lambda^0)$. Since the ground state is unique, then the measure $\sigma_{1^\perp}(\lambda)$ is continuous at the point $\lambda = \lambda_\Lambda^0$. Consequently, taking $a = \frac{1}{\sqrt{T}}$ we get that both terms in (130) tend to 0 as $T \to \infty$.

Using the same reasoning and formula (115) we obtain asymptotics (129). Lemma is proved completely.

We remaind that $\hat{\mathcal{P}}_{\Lambda,\infty,z}$ is a distribution on the space of trajectories of the process

$$\mathcal{G}_\Lambda \;=\; \{\gamma(t),\; t \in R^1\}$$

associated with Markov semigroup $\exp\{t\tilde{H}_\Lambda\}$. For any finite time intervals $t_0 < t_1 < \ldots < t_n$ and any bounded functions $f_0, f_1, \ldots, f_n$ on $\Gamma^0(\Lambda)$ the average over $\hat{\mathcal{P}}_{\Lambda,\infty,z}$ can be written as

$$\left\langle \prod_{i=0}^n f_i(\gamma(t_i)) \right\rangle_{\hat{\mathcal{P}}_{\Lambda,\infty,z}} \;= \qquad (131)$$

$$\int\limits_{(\Gamma^0(\Lambda))^n} f_n(\gamma_n) Q_{t_n - t_{n-1}}(\gamma_n, \gamma_{n-1}) \ldots f_1(\gamma_1) Q_{t_1 - t_0}(\gamma_1, \gamma_0) f_0(\gamma_0) \prod_{i=0}^n d\nu_z^\Lambda(\gamma_i),$$

where $Q_t(\gamma, \gamma')$ is the kernel of the operator $\exp\{t\tilde{H}_\Lambda\}$ in the space $L_2(\Gamma^0(\Lambda), \nu_z^\Lambda)$:

$$\left(\exp\{t\tilde{H}_\Lambda\}\; f\right)(\gamma) \;=\; \int\limits_{\Gamma^0(\Lambda)} Q_t(\gamma, \gamma') f(\gamma') d\nu_z^\Lambda(\gamma'). \qquad (132)$$

On the other hand, the average over $\mathcal{P}_{\Lambda,\infty,z}$, see (127), is calculated as the limit

$$\left\langle \prod_{i=0}^{n} f_i(\gamma(t_i)) \right\rangle_{\mathcal{P}_{\Lambda,\infty,z}} = \tag{133}$$

$$\lim_{T\to\infty} \frac{\int \prod_{i=0}^{n} f_i(\gamma(t_i)) \ \exp\left\{ -\alpha \int_{-T}^{T} U_\Lambda(\gamma(\tau))d\tau \right\} d\mathcal{P}_z^0}{Z_{\Lambda\times[-T,T]}}.$$

Using the Feynman-Kac representation (13) for the kernel of the semigroup e^{tH_Λ} and asymptotics (128) - (129) we can rewrite (133) as follows

$$\int\limits_{(\Gamma^0(\Lambda))^n} f_n(\gamma_n) \ \Psi_\Lambda(\gamma_n) \ R_{t_n-t_{n-1}}(\gamma_n,\gamma_{n-1}) \ e^{-\lambda_\Lambda^0(t_n-t_{n-1})} \ldots \tag{134}$$

$$\ldots e^{-\lambda_\Lambda^0(t_2-t_1)} f_1(\gamma_1) \ R_{t_1-t_0}(\gamma_1,\gamma_0) \ e^{-\lambda_\Lambda^0(t_1-t_0)} \ \Psi_\Lambda(\gamma_0) \ f_0(\gamma_0) \prod_{i=0}^{n} d\pi_z(\gamma_i).$$

It follows from (10) and (8) that

$$Q_t(\gamma_1,\gamma_2) \ = \ \frac{1}{\Psi_\Lambda(\gamma_1)} e^{-\lambda_\Lambda^0 t} \ R_t(\gamma_1,\gamma_2) \frac{1}{\Psi_\Lambda(\gamma_2)}, \quad d\nu_z^\Lambda(\gamma) \ = \ \Psi_\Lambda^2(\gamma) d\pi_z(\gamma),$$

and consequently, averages (131) and (134) are the same. Thus, all finite dimensional distributions for the measures $\mathcal{P}_{\Lambda,\infty,z}$ and $\hat{\mathcal{P}}_{\Lambda,\infty,z}$ coincide, and consequently, the measures are the same.

The first assertion of Theorem 4 is completely proved.

7.2 Second statement

Repeating arguments from the proof of Lemma 5 we get that for any bounded subset $G \subset K$ and bounded local function F_G the following limits exist and can be written as

$$\lim_{\Lambda\nearrow R^d} \langle F_G \rangle_{\mathcal{P}_{\Lambda,\infty,z}} \ = \ \langle F_G \rangle \ = \ f_G \ (F_G(\emptyset) + \tag{135}$$

$$\left. \sum_{m=1}^{\infty} \frac{1}{m!} \int\limits_{(\Upsilon^0(G)\setminus\emptyset)^m} \prod_{i=1}^{m} d\lambda^G(\eta_i) \int\limits_{(\Upsilon^0(K\setminus G))^m} \prod_{i=1}^{m} d\lambda^{K\setminus G}(\bar{\eta}_i) \ F_G(\cup \bar{\eta}_i) \prod_{i=1}^{m} K(\eta_i \cup \bar{\eta}_i) \right)$$

with

$$f_G \; = \; \exp\left\{ - \int\limits_{\Upsilon^0(G)\setminus\emptyset} \; \int\limits_{\Upsilon^0(K\setminus G)} K(\eta \cup \bar\eta) \; d\lambda^G(\eta) \; d\lambda^{K\setminus G}(\bar\eta) \right\} \; = \tag{136}$$

$$= \; \exp\left\{ - \int\limits_{\eta\in\Upsilon^0(K): \; \eta\cap G\neq\emptyset} K(\eta) \; d\lambda_z(\eta) \right\}.$$

By analogy with our reasoning in the proof of the first part of the theorem using corollary of Lemma 7 we get that limits (135) define a system of compatible probability distributions $\{\mathcal{P}^G_{\infty,z}\}$ on the space Υ', and thereby a limit distribution $\{\mathcal{P}_{\infty,z}\}$ on Υ' is defined. The limit distribution is invariant with respect to the space translations in R^d and w.r.t reflections in time. Moreover, this distribution meets the property of OS positivity. Really, for any local bounded function F dependent on the process as $t \geq 0$ we have

$$(\theta F \cdot F)_{\mathcal{P}_{\infty,z}} \; = \; \lim_{\Lambda\nearrow R^d} (\theta F, F)_{\mathcal{P}_{\Lambda,\infty z}} \; \geq \; 0, \tag{137}$$

since distributions $\mathcal{P}_{\Lambda,\infty,z}$ are the distributions of the Markov processes for any bounded $\Lambda \subset R^{d+1}$. Consequently, relation (137) is also valid for any function $F \in L_2(\Upsilon', \mathcal{P}_{\infty,z})$ which is also dependent on the process values as $t \geq 0$. Thus we construct the limit measure $\mathcal{P}_{\infty,z}$ and establish properties of this measure. Theorem 4 is completely proved.

8 Proof of Theorem 5

Let us consider strongly local functions $F_{M_i}, \; i = 1, 2$ depending on the process $\mathcal{G}$ (or $\mathcal{G}_\Lambda$) with bounded localization domain $M_i \subset R^{d+1}, \; i = 1, 2$. Denote by $G_i = G^{loc}_{M_i} \subset K$ a set of rods starting at M_i. Then the functions F_{M_i} can be considered as strongly local functions F_{G_i} on the space Υ' with the same localization domains $M_i, \; i = 1, 2$ correspondingly. We will use here the following formula for correlations (35), see [9]:

$$\langle F_{G_1} \cdot F_{G_2} \rangle_{\mathcal{P}_{\infty,z}} \; - \; \langle F_{G_1} \rangle_{\mathcal{P}_{\infty,z}} \; \langle F_{G_2} \rangle_{\mathcal{P}_{\infty,z}} \; = \tag{138}$$

$$f_{G_1} \cdot f_{G_2} \left(\left(e^{\hat\Delta(G_1,G_2)} - 1 \right) \; (F_{G_1}(\emptyset) \; F_{G_2}(\emptyset) + F_{G_1}(\emptyset) \; I_2 + F_{G_2}(\emptyset) \; I_1 + I_1 \; I_2) + \right.$$

$$e^{\hat{\Delta}(G_1,G_2)}\ I_{1,2}+F_{G_1}(\emptyset)\ \hat{I}_2+F_{G_2}(\emptyset)\ \hat{I}_1+I_1\ \hat{I}_2+I_2\ \hat{I}_1+\hat{I}_1\ \hat{I}_2\Big).$$

Here $f_{G_1},\ f_{G_2}$ are defined by (136),

$$\hat{\Delta}(G_1,G_2)\ =\ \int\limits_{\eta:\eta\cap G_1\neq\emptyset,\ \eta\cap G_2\neq\emptyset} K(\eta)\ d\lambda_z(\eta), \tag{139}$$

$$I_j\ =\ \sum_{n=0}^{\infty}\frac{1}{n!}\int\limits_{(\Upsilon^0(G_j)\setminus\emptyset\times\Upsilon^0(G_3))^n}\prod_{i=1}^{n}d\lambda^{G_j}(\eta^i_{G_j})\ d\lambda^{G_3}(\bar{\eta}^i_{G_3})\ F_{G_j}(\cup\eta^i_{G_j})\prod_{i=1}^{n}K(\eta^i_{G_j}\cup\bar{\eta}^i_{G_3})$$

with $G_3\ =\ K\backslash(G_1\cup G_2),\ j=1,2;$

$$\hat{I}_1\ =\ \sum_{n=0}^{\infty}\frac{1}{n!}\int F_{G_1}(\cup\eta^i_{G_1})\ \prod_{i=1}^{n}K(\eta^i_{G_1}\cup\eta^i_{G_2}\cup\eta^i_{G_3})\ \prod_{i=1}^{n}d\lambda^{G_1}(\eta^i_{G_1})\ d\lambda^{G_2}(\eta^i_{G_2})\ d\lambda^{G_3}(\eta^i_{G_3}),$$

where the integration is taken over all sets from n triplets

$$\left(\eta^i_{G_1},\ \eta^i_{G_2},\ \eta^i_{G_3}\right),\quad i=1,\dots,n,$$

such that $\eta^i_{G_1}\neq\emptyset$ for each $i=1,\dots,n$, and at least in one of the triplets: $\eta^i_{G_2}\neq\emptyset$. We can define $\hat{I}_2$ in the analogous way.

Further,

$$I_{1,2}\ =\ \sum_{n=0}^{\infty}\frac{1}{n!}\int\ F_{G_1}(\cup\eta^i_{G_1})\ F_{G_2}(\cup\eta^i_{G_2})$$

$$\prod_{i=1}^{n}K(\eta^i_{G_1}\cup\eta^i_{G_2}\cup\eta^i_{G_3})\ \prod_{i=1}^{n}d\lambda^{G_1}(\eta^i_{G_1})\ d\lambda^{G_2}(\eta^i_{G_2})\ d\lambda^{G_3}(\eta^i_{G_3}).$$

Here the integration is over all sets from n triplets

$$\left(\eta^i_{G_1},\ \eta^i_{G_2},\ \eta^i_{G_3}\right),\quad i=1,\dots,n,$$

such that at least in one of the triplets: $\eta^i_{G_1}\neq\emptyset$ and $\eta^i_{G_2}\neq\emptyset$.

Let us estimate $\hat{\Delta}(G_1,G_2)$ in the case when φ meets condition a) in (56), the case (56, b) can be studied in the same way. We can rewrite (63) as follows

$$|K(\eta)|\leq\sum_{\mathcal{T}}\ \prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}}\left(\frac{1}{1+|s_i-s_j|^{2m}}\right)^{\frac{1}{2}}\prod_{i\in V(\mathcal{T})}e^{-\frac{1}{2}l_i} \tag{140}$$

$$\prod_{\langle \xi_i,\xi_j\rangle \in \mathcal{T}} \left(1+|s_i-s_j|^{2m}\right)^{\frac{1}{2}} \prod_{i\in V(\mathcal{T})} e^{\frac{1}{2}l_i} \prod_{\langle \xi_i,\xi_j\rangle \in \mathcal{T}} \left|e^{-\alpha\,\Phi(\xi_i,\xi_j)}-1\right|,$$

with $\xi_i = ((s_i,t_i),l_i),\ i=1,\dots,n$. Then we have for any tree $\mathcal{T}$ in (140)

$$\prod_{\langle \xi_i,\xi_j\rangle \in \mathcal{T}} \left(\frac{1}{1+|s_i-s_j|^{2m}}\right)^{\frac{1}{2}} \prod_{i\in V(\mathcal{T})} e^{-\frac{1}{2}l_i} \ <$$

$$e^{-\sum\limits_{\langle \xi_i,\xi_j\rangle \in \mathcal{T}} m\ln(1+|s_i-s_j|)-\frac{1}{2}\sum\limits_{i\in V(\mathcal{T})} l_i} \ < \ e^{-diam\ \tilde{\eta}},$$

where $\tilde{\eta} = \cup\ \tilde{\xi}_i \subset R^{d+1}$ is a subset of R^{d+1} which is a union of all rods from configuration η, and the diameter of $\tilde{\eta}$ is calculated in the metrics (58, a). Thus,

$$|K(\eta)| \ < \ e^{-diam\ \tilde{\eta}} \sum_{\mathcal{T}} \prod_{\langle \xi_i,\xi_j\rangle \in \mathcal{T}} \left(\left(1+|s_i-s_j|^{2m}\right)^{\frac{1}{2}} \left|e^{-\alpha\,\Phi(\xi_i,\xi_j)}-1\right|\right) \prod_{i\in V(\mathcal{T})} e^{\frac{1}{2}l_i}.$$

Using bounds (55) - (56) on the potential φ we can apply here the above reasoning and finally get the following estimate

$$\int\limits_{\substack{\eta:\eta\cap G_1\neq\emptyset,\\ \eta\cap G_2\neq\emptyset}} K(\eta)\ d\lambda_z(\eta) \ < \ e^{-dist(M_1,M_2)}\ \max\{\upsilon_z(G_1),\upsilon_z(G_2)\}\ \hat{C}(\alpha,z), \quad (141)$$

where

$$\upsilon_z(G) \ = \ \int\limits_G z\ (-dp_x(l))dx,$$

and a constant $\hat{C}(\alpha,z)$ doesn't depend on G_1 and G_2. Thus, we have

$$\left|e^{\hat{\Delta}(G_1,G_2)}-1\right| \ < \ \hat{C}\ e^{-dist(M_1,M_2)}\ \max\{\upsilon_z(G_1),\upsilon_z(G_2)\}\ e^{\hat{C}\max\{\upsilon_z(G_1),\upsilon_z(G_2)\}}. \quad (142)$$

We estimate next I_1:

$$|I_1| \ = \ \max|F_{G_1}| \sum_{n=0}^{\infty} \frac{1}{n!} \left(\int\limits_{\Upsilon^0(G_1)\setminus\emptyset\times\Upsilon^0(G_3)} d\lambda^{G_1}(\eta_{G_1})\ d\lambda^{G_3}(\eta_{G_3})\ |K(\eta_{G_1}\cup\eta_{G_3})| \right)^n \ <$$

$$\max|F_{G_1}| \sum_{n=0}^{\infty} \frac{1}{n!} \left(\int\limits_{\eta:\eta\cap G_1\neq\emptyset} |K(\eta)|\, d\lambda(\eta) \right)^n < \max|F_{G_1}| \exp\{\hat{C} \upsilon_z(G_1)\}$$

with a constant $\hat{C}$. In the same way we can obtain the upper bound on I_2.

Let us estimate now $\hat{I}_1$:

$$|\hat{I}_1| < \max|F_{G_1}| \sum_{n=0}^{\infty} \frac{1}{n!} \int\limits_{\eta_{G_1}\neq\emptyset,\ \eta_{G_2}\neq\emptyset} K(\eta_{G_1} \cup \eta_{G_2} \cup \eta_{G_3}) \tag{143}$$

$$d\lambda^{G_1}(\eta_{G_1})\ d\lambda^{G_2}(\eta_{G_2})\ d\lambda^{G_3}(\eta_{G_3}) \left(\int\limits_{\eta:\eta\cap G_1\neq\emptyset} K(\eta)\ d\lambda(\eta) \right)^{n-1} .$$

The first integral in (143) can be estimated in the same manner as above, and we get

$$|\hat{I}_1| < e^{-dist(M_1,M_2)} \max\{\upsilon_z(G_1), \upsilon_z(G_2)\} \max|F_{G_1}|\ e^{\hat{C}\upsilon_z(G_1)}.$$

The analogous estimate is valid also for $\hat{I}_2$ and $I_{1,2}$. Finally representation (136) implies

$$|f_G| < \exp\left\{ \int\limits_{\eta:\eta\cap G\neq\emptyset} |K(\eta)|\ d\lambda_z(\eta) \right\} < \exp\{\hat{C}\ \upsilon_z(G)\}.$$

Since $\upsilon_z(G) = z|M|$ in the case $G = G_M^{loc}$, then after substitution all above estimates to (138) we get the main estimate (35) of Theorem 4.

8.1 Proof of Corollary from Theorem 5

We study here the generator h of the time translations in the physical space $\mathcal{H}$ and prove that the operator h has a spectral gap. Let $\mu = \sup \sigma\,(h|_{\mathcal{H}^\perp})$ be the supremum of a spectrum of the restriction of the operator h to the invariant subspace $\mathcal{H}^\perp$ containing vectors from $\mathcal{H}$ orthogonal to vector e. We denote by $\hat{\mathcal{H}} \subset \mathcal{H}$ a dense set in $\mathcal{H}$ such that classes of elements from $\hat{\mathcal{H}}$

contain strongly local bounded functions on $\hat{\Upsilon}$. For any $\varepsilon > 0$ there exists an element $\varphi \in \hat{\mathcal{H}}$ such that

$$1)\ (\varphi, e) = 0, \quad 2)\ (\varphi, \varphi) = 1, \quad 3)\ \sigma_\varphi(\mu - \varepsilon, \mu) > \frac{1}{2}, \tag{144}$$

where $\sigma_\varphi(\Delta) = (E_h(\Delta)\varphi, \varphi)_{\mathcal{H}}$ is the spectral measure of the element φ, and $E_h(\Delta)$ is the resolution of identity of the operator h. Condition 3) implies that

$$(e^{th}\varphi, \varphi)_{\mathcal{H}} \ = \ \int\limits_{-\infty}^{\mu} e^{\lambda t}\ d\sigma_\varphi(\lambda) \ > \frac{1}{2} e^{(\mu-\varepsilon)t}. \tag{145}$$

Let $\Phi \in \mathcal{H}_+$ is a strongly local bounded function (not a constant) such that the class $[\Phi]$ of Φ is the same as φ. Then conditions (144) is rewritten as

$$1)\ \langle \Phi \rangle_{\mathcal{P}_{\infty,z}} = 0, \quad 2)\ \langle \theta\ \Phi \cdot \Phi \rangle_{\mathcal{P}_{\infty,z}} = 1, \quad 3)\ \langle \theta\ U_t\ \Phi \cdot \Phi \rangle_{\mathcal{P}_{\infty,z}} \ > \ \frac{1}{2} e^{(\mu-\varepsilon)t}. \tag{146}$$

Localization domain $M_1 \neq \emptyset$ of the function Φ is in the right half-space $R^{d+1}_+ = R^d \times R^1_+$ and localization domain M_2 of the function $\theta\ U_t \Phi$ is in the left half-space $R^{d+1}_- = R^d \times R^1_-$. Moreover the distance in the metrics (34) between M_1 and M_2 is not less then $\frac{t}{2}$. Then using the result of Theorem 4 we have

$$(\theta\ U_t\ \Phi,\ \Phi)_{\mathcal{H}} \ = \ \langle \theta\ U_t\ \Phi \cdot \Phi \rangle_{\mathcal{P}_{\infty,z}} \ < \ c_1\ e^{-\frac{t}{2}}, \tag{147}$$

with a constant c_1 doesn't depending on t. Comparing (145) with (147) it is easy to see that

$$(\mu - \varepsilon)\ t \ < \ -\frac{1}{2}\ t \ + \ c_2 \tag{148}$$

with an absolute constant c_2. Since inequality (148) holds for any $\varepsilon > 0$ and any $t > 0$, we get $\mu \leq -\frac{1}{2}$. That means that the operator h has a spectral gap and the unique ground state.

9 Attachment

9.1 Proof of Lemma 1

Let $M \subset R^{d+1}$ be a bounded set. Without loss of generality we can take $M = \Lambda \times I$, where $\Lambda \subset R^d$ is a bounded domain in R^d and $I = [T_1, T_2] \subset R^1$

is a segment. Then for a.e. configurations η of rods a number of rods with origins inside of M is finite. For the proof, it remained to show that the mean value of a number of rods from η with origins $x = (s, t)$ at the "past" to M (i.e. with $t < T_1$) intersecting M is also finite.

We denote $G_{M_n}^{(n)} \subset K$ a set of rods beginning at $M_n = \Lambda \times I_n$ where $I_n = (T_1 - (n+1), T_1 - n)$ with a lenght not less then n, and let $G_{M_n}^{M} \subset K$ be a set of rods with origins in M_n intersecting M. It is clear that $G_{M_n}^{M} \subset G_{M_n}^{(n)}$ and $\Upsilon(G_{M_n}^{M}) \subset \Upsilon(G_{M_n}^{(n)})$. On the other hand, the probability that a rod $\xi \in G_{M_n}^{loc}$ belongs to a configuration $\eta_n \in \Upsilon(G_{M_n}^{(n)})$ equals to e^{-n}, consequently the set of origins of rods from $\Upsilon(G_{M_n}^{(n)})$ forms a Poisson field in M_n with intensity ze^{-n}. We denote $\eta_n^M \subseteq \eta_n$ a sub-configuration of η_n such that $\eta_n^M \in \Upsilon(G_{M_n}^{M})$. Averaging over $\mathcal{P}_z^0$ we get

$$\langle |\eta_n^M| \rangle_{\mathcal{P}_z^0} \ < \ \langle |\eta_n| \rangle_{\mathcal{P}_z^0} \ = \ z|\Lambda| e^{-n}.$$

Thus the mean value of a number of rods intersecting M with beginnings at the "past" of M can be bounded from above by

$$z|\Lambda| \sum_{n=0}^{\infty} e^{-n} \ < \ \infty.$$

That means that configurations with infinite number of rods intersecting M form a set of zero measure. Since the space R^{d+1} can be covered by a countable family of bounded sets of the form $M = \Lambda \times I$, then almost all configurations of rods are locally finite.

Lemma is proved.

9.2 Proof of Proposition 1

Let us consider a quadratic form for any $F \in \mathcal{H}_+$ and $t > 0$:

$$q_F(t) \ = \ (F, \ U_t \ F)_+.$$

Using Caugchi-Buniakovskyi-Schwarz inequality n times and equality $(U_t F, F)_+ = (F, U_t F)_+$ we get

$$q_F(t) \ \leq \ (F,F)_+^{\frac{1}{2}} \ (U_t F, U_t F)_+^{\frac{1}{2}} \ = \ (F,F)_+^{\frac{1}{2}} \ (F, U_{2t} F)_+^{\frac{1}{2}} \ \leq \qquad (149)$$

$$(F,F)_+^{\frac{1}{2}+\frac{1}{4}} \ (F, U_{4t} F)_+^{\frac{1}{4}} \ \leq \ldots \ \leq \ (F,F)_+^{1-\frac{1}{2^n}} \ (F, U_{2^n t} F)_+^{\frac{1}{2^n}}.$$

For any $t > 0$ and any n

$$(F, U_{2^n t} F)_+ \ = \ (\theta F, U_{2^n t} F)_{L_2(\Omega,\mathcal{P})} \ \leq \ \|F\|^2_{L_2}.$$

Since $q_F(t)$ doesn't depend on n, we take a limit in (149) as $n \to \infty$ and obtain for any t

$$q_F(t) \ \leq \ (F, F)_+.$$

Finally,

$$(U_t F, U_t F)_+ \ = \ (F, U_{2t} F)_+ \ = \ q_F(2t) \ \leq \ (F, F)_+.$$

9.3 Proof of Lemma 4

Note, that by virtue of (51)-(52) the measure in the right hand side of (53) is the probabilistic one. Under decomposition $G^0_{\Lambda,T} = G_1 \cup G_2$ on two non-itersecting sets this measure by (49) could be written as a product of two probabilistic measures:

$$\lambda^{G_1} \ e^{-S(G_1)} \ \times \ \lambda^{G_2} \ e^{-S(G_2)}$$

with $S(G) = \int\limits_G \xi(x) \ p_x(l) \ dl \ dx$. For the probability that the number of rods $|\eta_{G_i}|$ in the configuration $\eta_{G_i} \subset \Upsilon^0(G_i)$ equals k, we get:

$$\Pr(|\eta_{G_i}| = k) \ = \ \frac{1}{k!}(S(G_i))^k e^{-S(G_i)},$$

i.e. the probability has a Poisson form. That means that the measure in (53) is the distribution of the Poisson field of the form $\Pi(M, \zeta, \{p_x\})$, where $M, \zeta, \{p_x\}$ are the same as in formula (42).

9.4 Proof of Lemma 5

Let us start with the case when $\varphi \geq 0$. Inequality (63) is evident for a set of nodes $\eta = \{\xi_1, \xi_2, \xi_3\}$. We assume that (63) holds for all sets $\eta^{(m)} = \{\xi_1, \dots, \xi_m\}$ of nodes with $2 \leq m \leq n$, and prove the statement of the Lemma for a vertex set $V(\sigma) = \eta^{(n+1)} = \{\xi_0, \dots, \xi_n\}$ using the induction assumption.

We note first that κ^T_σ defined by (61) can be rewritten for any given connected graph σ with a vertex set $V(\sigma) = \eta^{(n+1)}$ as

$$\kappa^T_\sigma = \prod_{i=1}^{k} \kappa^T_{\sigma_i} \prod_{\xi_j \in m_i} \left(e^{-\alpha\, \Phi^T(\xi_0, \xi_j)} - 1 \right). \tag{150}$$

Here $\sigma_i,\ i=1,\ldots,k$ are connected subgraphs of the graph σ with a vertex set $V(\sigma_i)\subseteq\eta^{(n)}=\{\xi_1,\ldots,\xi_n\}$, such that $\sigma_1,\ldots,\sigma_k$ form a decomposition of the rest of σ after removing all edges incident to the node ξ_0, and $m_i\subseteq V(\sigma_i)$ is a subset of $V(\sigma_i)$ governing a subset of incident to ξ_0 edges with ends from $V(\sigma_i)$.

Then using (150) we have

$$\sum\nolimits_{\sigma}^{(\eta^{(n+1)})}\kappa_\sigma^T \;=\; \sum_{k=1}^{n}\sum\nolimits_{\{\eta_1,\ldots,\eta_k\}}^{(\eta^{(n)})}\prod_{i=1}^{k}\left(\sum_{\sigma_i:\ V(\sigma_i)=\eta_i}\kappa_{\sigma_i}^T\right.$$

$$\left.\sum_{m_i\subseteq\eta_i,m_i\neq\emptyset}\prod_{\xi_j\in m_i}\left(e^{-\alpha\,\Phi^T(\xi_0,\xi_j)}-1\right)\right)\;\le$$

$$\sum_{k=1}^{n}\sum\nolimits_{\{\eta_1,\ldots,\eta_k\}}^{(\eta^{(n)})}\prod_{i=1}^{k}\left(\sum_{\mathcal{T}_i:\ V(\mathcal{T}_i)=\eta_i}\ \prod_{\langle\xi_\alpha,\xi_\beta\rangle\in\mathcal{T}_i}\left|e^{-\alpha\Phi^T(\xi_\alpha,\xi_\beta)}-1\right|\sum_{\xi_j\in\eta_i}\left|e^{-\alpha\,\Phi^T(\xi_0,\xi_j)}-1\right|\right), \tag{151}$$

where in (151) the internal sum is taken over all trees $\mathcal{T}_i$ with vertex set η_i. In the last bound we applied the induction assumption, the identity

$$\sum_{m\subseteq\eta,m\neq\emptyset}\ \prod_{\xi\in m}\left(e^{-\alpha\,\Phi^T(\xi_0,\xi)}-1\right)\;=\;e^{-\alpha\sum\limits_{\xi\in\eta}\Phi^T(\xi_0,\xi)}-1$$

and the following inequlity

$$\left|e^{-\alpha\sum\limits_{\xi\in\eta}\Phi^T(\xi_0,\xi)}-1\right|\;\le\;\sum_{\xi\in\eta}\left|e^{-\alpha\Phi^T(\xi_0,\xi)}-1\right| \tag{152}$$

which is valid for $\Phi^T\ge 0$.

Connecting all possible trees from $\mathcal{T}_i,\ i=1,\ldots,k$ with all possible edges $(\xi_0,\xi_j),\ \xi_j\in\eta_i$ from the last sum in (151) we obtain all possible trees $\mathcal{T}$ with $V(\mathcal{T})=\eta^{(n+1)}$, consequently the sum in (151) is the same as

$$\sum\nolimits_{\mathcal{T}}^{(\eta^{(n+1)})}\prod_{\langle\xi_i,\xi_j\rangle\in\mathcal{T}}\left|\left(e^{-\alpha\Phi^T(\xi_i,\xi_j)}-1\right)\right|.$$

If φ is a general stable potential, then the Ruelle condition for n-points configuration γ_n

$$U(\gamma_n) \equiv \sum_{\{x,y\}\subset\gamma_n} \varphi(x-y) \ \geq \ -\frac{1}{2}Bn$$

can be rewritten as a corresponding condition on rod configurations $\eta \in \Upsilon^0(G^0_{\Lambda,T})$:

$$\sum_{\{\xi,\xi'\}\subset\eta} \Phi(\xi,\xi') \ \geq \ -\frac{1}{2}B\sum_{\xi\in\eta} l(\xi),$$

where $l(\xi)$ is the length of the rod ξ. This estimate implies existence of such $\xi_0 \in \eta$ that

$$\sum_{\xi\in\eta\setminus\xi_0} \Phi(\xi_0,\xi) \ \geq \ -B\ l(\xi_0). \tag{153}$$

We take in the configuration $\eta^{(n+1)}$ such rod $\xi_0 \in \eta^{(n+1)}$ which was indicated in inequality (153), and prove the modification of inequality (152). Let us consider a decomposition of η into two sub-configurations: $\eta = \eta_+ \cup \eta_-$ with

$$\eta_+ = \{\xi \in \eta : \Phi^T(\xi_0,\xi) \geq 0\}, \quad \eta_- = \{\xi \in \eta : \Phi^T(\xi_0,\xi) < 0\}.$$

Then using inequalities (153) and (152) (the last one holds for $\Phi^T \geq 0$) we have

$$\left| e^{-\alpha \sum\limits_{\xi\in\eta} \Phi^T(\xi_0,\xi)} - 1 \right| \ \leq$$

$$\left| e^{-\alpha \sum\limits_{\xi\in\eta_+\cup\eta_-} \Phi^T(\xi_0,\xi)} - e^{-\alpha \sum\limits_{\xi\in\eta_+} \Phi^T(\xi_0,\xi)} \right| + \left| e^{-\alpha \sum\limits_{\xi\in\eta_+} \Phi^T(\xi_0,\xi)} - 1 \right| \ \leq$$

$$e^{-\alpha \sum\limits_{\xi\in\eta} \Phi^T(\xi_0,\xi)} \left| 1 - e^{\alpha \sum\limits_{\xi\in\eta_-} \Phi^T(\xi_0,\xi)} \right| + \left| e^{-\alpha \sum\limits_{\xi\in\eta_+} \Phi^T(\xi_0,\xi)} - 1 \right| \ \leq$$

$$e^{\alpha B l(\xi_0)} \sum_{\xi\in\eta_-} \left| 1 - e^{\alpha\Phi^T(\xi_0,\xi)} \right| + \sum_{\xi\in\eta_+} \left| e^{-\alpha\Phi^T(\xi_0,\xi)} - 1 \right| \ \leq$$

$$e^{\alpha B l(\xi_0)} \sum_{\xi\in\eta} \left(1 - e^{-\alpha\left|\Phi^T(\xi_0,\xi)\right|} \right). \tag{154}$$

Repeating above reasoning under the induction assumption and revised estimate (154) we obtain estimate (62) in the general case of a stable potential φ.

Lemma is proved.

Acknowledgements.

The authors gratefully acknowledge the financial support of the SFB 701, Bielefeld University. R.M. and E.Zh. acknowledge the partial financial support of the RFBR grant 05-01-00449 and the CRDF grant RUM1-2693-MO-05.

References

[1] S. Albeverio, Yu.G. Kondratiev, M. Rockner, Analysis and geometry on configuration spaces, J. Funct. Anal. 154, 1998, p. 444-500.

[2] N. Angelescu, R.A. Minlos, V.A. Zagrebnov, The one-particle branch of the energy spectrum of weakly interacting quantum lattice fields with values on the two-dimensional sphere, J. Math. Phys., Vol. 41, p. 2-23, 2000.

[3] Berezansky, Yu.M., Kondratiev, Yu.G.: Spectral methods in infinite dimensional analysis, Vol. 1,2, Kluwer Academic Publisher, Dordrecht - Boston - London (1995)

[4] J.L. Doob, Stochastic processes, John Wiley and Sons, 1967.

[5] M. Duneau, D. Jagolnitzer, B. Souillard, Decrease properties of truncated correlation functions and analyticity properties for classical lattices and continuous systems, Comm. Math. Phys., 31, 1973, p. 191-208.

[6] D. Glimm, A. Jaffe, Quantum Physics. A functional integral point of view, Springer-Verlag, 1981.

[7] O. Kalenberg, Random measures. New York: Academic Press, 1983.

[8] Yu. Kondratiev, E. Lytvynov: Glauber dynamics of continuous particle systems, Ann. Inst. H. Poincare - PR 41, 2005, p. 685-702.

[9] Yu.G. Kondratiev, R.A. Minlos, M. Rockner, G.V. Shchepanuk, Exponential mixing for classical continuous systems, Canadian Math. Society, Conference proceedings, Vol. 20, 2000, p. 243-253.

[10] Yu.G. Kondratiev, R.A. Minlos, E.A. Zhizhina, Lower branches of the spectrum of Hamiltonians for infinite quantum system with compact spin, Trans. of Moscow Mathem. Society 1998, Vol.60, p. 259-302.

[11] J. Lorinczi, R.A. Minlos, H. Spohn, The infrared behaviour in Nelson's model of a quantum particle coupled to a massless scalar field, Ann. Henri Poincare 3, 2002, p. 269-295.

[12] V.A. Malyshev, R.A. Minlos, Gibbs random fields, Kluwer Academic Publishers, 1991.

[13] R.A. Minlos, Limiting Gibbs distribution, Functs. Analis i ego priloshenia, 1 (2), 1967, p. 60-73.

[14] R.A. Minlos, S.K. Pogosyan, Estimates on the Ursell functions, group functions and their derivatives, Teoretich. i matematich. fisika, 31 (2), 1977, p. 199-213.

[15] O. Ore, Theory of graphs, AMS Colloquium Publ. 38, 1962.

[16] M. Reed, B. Simon, Methods of modern mathematical physics, Vol. 1-4, Acad. press, 1978.

[17] Ruelle, D.: Statistical Mechanics. Rigorous Results, Benjamins, Amsterdam , 1969.

[18] B. Simon, The $P(\varphi)_2$ Euclidean Quantum Field Theory, Princeton University Press, 1974.

[19] B. Simon, Functional integration and quantum physics, Acad. Press, 1979.

[20] B. Simon, The statistical mechanics of lattice gas, Vol.1, Princeton University Press, 1993.

[21] I. D. Surgailis, On multiple Poisson stochastic integrals and associated Markov semigroups, Probability and mathematical statistics, 3 (1984), 217-239. II. D. Surgailis, On Poisson multiple stochastic integrals and

associated equilibrium Markov processes, in Theory and Applic. of Random Fields, (Bangalore, 1982), Lecture Notes in Control and Inform. Sci., 49, Springer, (1983), 233-248.