

# ERGODICITY OF NON-EQUILIBRIUM GLAUBER DYNAMICS IN CONTINUUM

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## Abstract

We study asymptotic properties of the continuous Glauber dynamics with unbounded death and constant birth rates. In particular, an information about the spectrum location for the symbol of the Markov generator is obtained. The latter fact is used for the proof of the ergodicity of this process. We show that the speed of convergence to the equilibrium is exponential.

*Keywords:* Configuration space; continuous Glauber dynamics; ergodicity of Markov process.

## 1 Introduction

The continuous Glauber type dynamics may be characterized as birth-and-death Markov processes on the configuration space (CS) in continuum with the given grand canonical Gibbs equilibrium states as the symmetrizing measures. The Markov generators of these processes are related to the (non-local) Dirichlet forms of Gibbs measures. The latter fact gives the possibility to construct the so-called equilibrium Glauber dynamics associated with these forms [18]. There are many

possibilities to choose a particular form of the square field expression inside of the Dirichlet forms. Each of them leads to the corresponding birth and death intensities in the generator of the Glauber dynamics. For example, one may consider a constant death rate, which means that all information about the interaction in the system is included in the birth rate ( $G^+$  Glauber dynamics). Another extremal possibility is to take a constant birth rate and to assure the reversibility of the dynamics via proper death coefficient ( $G^-$  Glauber dynamics). In the case of  $G^+$  equilibrium dynamics and a positive interaction potential, there exists a spectral gap for the generator in the high temperature and low density regime [3], [18], [32]. This gives the exponential  $L^2$  ergodicity for the corresponding Markov semigroup.

Coming to the problem of the non-equilibrium Glauber dynamics, we note that only recently were constructed Markov processes with some classes of initial distributions (i.e., Markov functions) for  $G^+$  [17] and  $G^-$  [16] dynamics. These papers use a constructive approach to the dual Kolmogorov equation describing the evolution of the initial distributions in the Glauber stochastic dynamics. Let us stress that in the infinite particle case, the class of admissible initial conditions should be considered as an essential parameter in the study of dynamical properties. Depending on the initial conditions, stochastic dynamics may have very different types of behavior including the possibility to explode in a finite time. Roughly speaking, a choice of initial states defines the level of deviation from the equilibrium dynamics.

In the present paper we analyze ergodic properties of such non-equilibrium random evolutions. Namely, we consider the case of  $G^-$  stochastic dynamics for interacting potentials which satisfy stability and strong integrability conditions (but admit a possible negative part). The main result concerning the ergodicity is stated in Theorem 3.2. We show that under the natural restrictions on the parameters of the system the time evolution for a class of initial measures converges to the invariant Gibbs measure. More precisely, we need to consider the equilibrium state in the high temperature and low density regime to assure the uniqueness of the limiting Gibbs measure. Then depending on these parameters we define explicitly the set of admissible initial states. Let us stress that this set forms a ball (in a proper metric) in the space of all probability measures on the CS. This ball includes the invariant Gibbs measure as well as all probability measures on the CS with the common Ruelle bounds. The convergence of measures on the CS is defined in the sense of their correlation functions convergence.

We show the exponential rate of such convergence in the Ruelle type norm on correlation functions.

## 2 Foundations

We consider the Euclidian space  $\mathbb{R}^d$ . By  $\mathcal{B}(\mathbb{R}^d)$  we denote the family of all Borel sets in  $\mathbb{R}^d$ .  $\mathcal{B}_b(\mathbb{R}^d)$  denotes the system of all sets in  $\mathcal{B}(\mathbb{R}^d)$  which are bounded.

The space of *n-point configuration* is

$$\Gamma_0^{(n)} = \Gamma_{0, \mathbb{R}^d}^{(n)} := \left\{ \eta \subset \mathbb{R}^d \mid |\eta| = n \right\}, \quad n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\},$$

where  $|A|$  denotes the cardinality of the set  $A$ .

The space  $\Gamma_\Lambda^{(n)} = \Gamma_{0, \Lambda}^{(n)}$  for  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  is defined analogously to the space  $\Gamma_0^{(n)}$ . As a set  $\Gamma_0^{(n)}$  is equivalent to the symmetrization of

$$\widetilde{(\mathbb{R}^d)^n} = \left\{ (x_1, \dots, x_n) \in (\mathbb{R}^d)^n \mid x_k \neq x_l \text{ if } k \neq l \right\},$$

i.e. to  $\widetilde{(\mathbb{R}^d)^n}/S_n$ , where  $S_n$  is the permutation group of  $\{1, \dots, n\}$ . Hence, one can introduce the corresponding topology and Borel  $\sigma$ -algebra, which we denote by  $\mathcal{O}(\Gamma_0^{(n)})$  and  $\mathcal{B}(\Gamma_0^{(n)})$ , respectively.

The space of *finite configurations*

$$\Gamma_0 := \bigsqcup_{n \in \mathbb{N}_0} \Gamma_0^{(n)}$$

is equipped with the topology  $\mathcal{O}(\Gamma_0)$  of disjoint union. Let  $\mathcal{B}(\Gamma_0)$  denotes the corresponding Borel  $\sigma$ -algebra.

A set  $B \in \mathcal{B}(\Gamma_0)$  is called *bounded* if there exists  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  and  $N \in \mathbb{N}$  such that  $B \subset \bigsqcup_{n=0}^N \Gamma_\Lambda^{(n)}$ .

The *configuration space*

$$\Gamma := \left\{ \gamma \subset \mathbb{R}^d \mid |\gamma \cap \Lambda| < \infty, \text{ for all } \Lambda \in \mathcal{B}_b(\mathbb{R}^d) \right\}$$

is equipped with the *vague topology*  $\mathcal{O}(\Gamma)$ . It is a Polish space (see e.g. [15]).  $\mathcal{B}(\Gamma)$  denotes the corresponding Borel  $\sigma$ -algebra. The *filtration* on  $\Gamma$  with a base set  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  is given by

$$\mathcal{B}_\Lambda(\Gamma) := \sigma \left( N_{\Lambda'} \mid \Lambda' \in \mathcal{B}_b(\mathbb{R}^d), \Lambda' \subset \Lambda \right),$$

where  $N_\Lambda : \Gamma_0 \rightarrow \mathbb{N}_0$  is such that  $N_\Lambda(\eta) := |\eta \cap \Lambda|$ . For short we write  $\eta_\Lambda := \eta \cap \Lambda$ .

For every  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  the *projection*  $p_\Lambda : \Gamma \rightarrow \Gamma_\Lambda := \bigsqcup_{n \geq 0} \Gamma_\Lambda^{(n)}$  is defined as

$$p_\Lambda(\gamma) := \gamma_\Lambda$$

One can show that  $\Gamma$  is the projective limit of the spaces  $\{\Gamma_\Lambda\}_{\Lambda \in \mathcal{B}_b(\mathbb{R}^d)}$  w.r.t. this projections.

In the sequel we will use the following classes of function on  $\Gamma_0$ :

- $L^0(\Gamma_0)$  - the set of all *measurable functions* on  $\Gamma_0$ ;
- $L_{\text{ls}}^0(\Gamma_0)$  - the set of measurable *functions with local support*, i.e.  $G \in L_{\text{ls}}^0(\Gamma_0)$  if there exists  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  such that  $G \upharpoonright_{\Gamma_0 \setminus \Gamma_\Lambda} = 0$ ;
- $L_{\text{bs}}^0(\Gamma_0)$  - the set of measurable *functions with bounded support*, i.e.  $G \in L_{\text{bs}}^0(\Gamma_0)$  if there exists  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  and  $N \in \mathbb{N}$  such that  $G \upharpoonright_{\Gamma_0 \setminus \bigsqcup_{n=0}^N \Gamma_\Lambda^{(n)}} = 0$ ;
- $B(\Gamma_0)$  - the set of *bounded measurable functions* ;
- $B_{\text{bs}}(\Gamma_0)$  - the set of *bounded functions with bounded support*.

On  $\Gamma$  we consider the set of *cylinder functions*  $\mathcal{FL}^0(\Gamma)$ , i.e. the set of all measurable functions  $G \in L^0(\Gamma)$  which are measurable w.r.t.  $\mathcal{B}_\Lambda(\Gamma)$  for some  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ . These functions are characterized by the following relation:

$$F(\gamma) = F \upharpoonright_{\Gamma_\Lambda}(\gamma_\Lambda).$$

Those cylinder functions which are measurable w.r.t.  $\mathcal{B}_\Lambda(\Gamma)$  for fixed  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  we will denote by  $\mathcal{FL}^0(\Gamma, \mathcal{B}_\Lambda(\Gamma))$ .

Next we would like to describe some facts from the harmonic analysis on the configuration spaces based on [13].

The following mapping between functions on  $\Gamma_0$  and functions on  $\Gamma$  plays the key role in our further considerations:

$$KG(\gamma) := \sum_{\xi \in \gamma} G(\xi), \quad G \in L_{\text{ls}}^0(\Gamma_0) \quad \gamma \in \Gamma,$$

see e.g. [23, 24]. The summation in the latter expression is taken over all finite subconfigurations of  $\gamma$ , which is denoted by symbol  $\xi \in \gamma$ .

$K$ -transform is linear, positivity preserving, and invertible, with

$$K^{-1}F(\eta) := \sum_{\xi \subset \eta} (-1)^{|\eta \setminus \xi|} F(\xi), \quad F \in \mathcal{FL}^0(\Gamma) \quad \eta \in \Gamma_0. \quad (1)$$

The map  $K$ , as well as the map  $K^{-1}$ , can be extended to more wide classes of functions. For details and further properties of the map  $K$  see, e.g. [13].

One can introduce a *convolution*

$$\begin{aligned} \star : L^0(\Gamma_0) \times L^0(\Gamma_0) &\rightarrow L^0(\Gamma_0) & (2) \\ (G_1, G_2) &\mapsto (G_1 \star G_2)(\eta) \\ := \sum_{(\xi_1, \xi_2, \xi_3) \in \mathcal{P}_\emptyset^3(\eta)} & G_1(\xi_1 \cup \xi_2) G_2(\xi_2 \cup \xi_3), \end{aligned}$$

where  $\mathcal{P}_\emptyset^3(\eta)$  denotes the set of all partitions  $(\xi_1, \xi_2, \xi_3)$  of  $\eta$  in 3 parts, i.e., all triples  $(\xi_1, \xi_2, \xi_3)$  with  $\xi_i \subset \eta$ ,  $\xi_i \cap \xi_j = \emptyset$  if  $i \neq j$ , and  $\xi_1 \cup \xi_2 \cup \xi_3 = \eta$ .

It has the property that for  $G_1, G_2 \in L_{\text{ls}}^0(\Gamma_0)$

$$K(G_1 \star G_2) = KG_1 \cdot KG_2.$$

Due to this convolution we can interpret the  $K$ -transform as the Fourier transform in configuration space analysis, see also [2].

Let  $\mathcal{M}^1(\Gamma)$  be the set of all probability measures on  $\mathcal{B}(\Gamma)$  and let  $\mathcal{M}_{\text{fm}}^1(\Gamma)$  be the set of all probability measures  $\mu$  which have *finite local moments* of all orders, i.e.

$$\int_{\Gamma} |\gamma_\Lambda|^n \mu(d\gamma) < +\infty$$

for all  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$  and  $n \in \mathbb{N}_0$ .

A measure  $\rho$  on  $(\Gamma_0, \mathcal{B}(\Gamma_0))$  is called *locally finite* if  $\rho(A) < \infty$  for all bounded sets  $A$  from  $\mathcal{B}(\Gamma_0)$ . The set of such measures is denoted by  $\mathcal{M}_{\text{lf}}(\Gamma_0)$ .

A measure  $\rho \in \mathcal{M}_{\text{lf}}(\Gamma_0)$  is called *positive definite* if

$$\int_{\Gamma_0} (G \star \overline{G})(\eta) \rho(d\eta) \geq 0, \quad \forall G \in B_{\text{bs}}(\Gamma_0),$$

where  $\overline{G}$  is a complex conjugate of  $G$ .

A measure  $\rho$  is called *normalized* if and only if  $\rho(\{\emptyset\}) = 1$ .

One can define a transform  $K^* : \mathcal{M}_{\text{fm}}^1(\Gamma) \rightarrow \mathcal{M}_{\text{lf}}(\Gamma_0)$ , which is *dual* to the  $K$ -transform, i.e., for every  $\mu \in \mathcal{M}_{\text{fm}}^1(\Gamma)$ ,  $G \in \mathcal{B}_{\text{bs}}(\Gamma_0)$  we have

$$\int_{\Gamma} KG(\gamma) \mu(d\gamma) = \int_{\Gamma_0} G(\eta) (K^* \mu)(d\eta).$$

The measure  $\rho_\mu := K^*\mu$  is called the *correlation measure* of  $\mu$ . As shown in [13] for  $\mu \in \mathcal{M}_{\text{fm}}^1(\Gamma)$  and any  $G \in L^1(\Gamma_0, \rho_\mu)$  the series

$$KG(\gamma) := \sum_{\eta \in \gamma} G(\eta), \quad (3)$$

is  $\mu$ -a.s. absolutely convergent. Furthermore,  $KG \in L^1(\Gamma, \mu)$  and

$$\int_{\Gamma_0} G(\eta) \rho_\mu(d\eta) = \int_{\Gamma} (KG)(\gamma) \mu(d\gamma). \quad (4)$$

Fix a non-atomic and locally finite measure  $\sigma$  on  $(\mathbb{R}^d, \mathcal{B}(\mathbb{R}^d))$ . For any  $n \in \mathbb{N}$  the product measure  $\sigma^{\otimes n}$  can be considered by restriction as a measure on  $\widetilde{(\mathbb{R}^d)^n}$  and hence on  $\Gamma_0^{(n)}$ . The measure on  $\Gamma_0^{(n)}$  we denote by  $\sigma^{(n)}$ .

The *Lebesgue-Poisson measure*  $\lambda_{z\sigma}$  on  $\Gamma_0$  is defined as

$$\lambda_{z\sigma} := \sum_{n=0}^{\infty} \frac{z^n}{n!} \sigma^{(n)}.$$

Here  $z > 0$  is the so-called activity parameter. The restriction of  $\lambda_{z\sigma}$  to  $\Gamma_\Lambda$  will be also denoted by  $\lambda_{z\sigma}$ . We write  $\lambda_z$  instead of  $\lambda_{z\sigma}$ , if the measure  $\sigma$  is considered to be fixed.

The *Poisson measure*  $\pi_{z\sigma}$  on  $(\Gamma, \mathcal{B}(\Gamma))$  is given as the projective limit of the family of measures  $\{\pi_{z\sigma}^\Lambda\}_{\Lambda \in \mathcal{B}_b(\mathbb{R}^d)}$ , where  $\pi_{z\sigma}^\Lambda$  is the measure on  $\Gamma_\Lambda$  defined by  $\pi_{z\sigma}^\Lambda := e^{-z\sigma(\Lambda)} \lambda_{z\sigma}$ .

A measure  $\mu \in \mathcal{M}_{\text{fm}}^1(\Gamma)$  is called *locally absolutely continuous w.r.t.*  $\pi_{z\sigma}$  iff  $\mu_\Lambda := \mu \circ p_\Lambda^{-1}$  is absolutely continuous with respect to  $\pi_{z\sigma}^\Lambda = \pi_{z\sigma} \circ p_\Lambda^{-1}$  for all  $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ . In this case,  $\rho_\mu := K^*\mu$  is absolutely continuous w.r.t  $\lambda_{z\sigma}$ . Let  $k_\mu : \Gamma_0 \rightarrow \mathbb{R}_+$  be the corresponding Radon-Nikodym derivative, i.e.

$$k_\mu(\eta) := \frac{d\rho_\mu}{d\lambda_{z\sigma}}(\eta), \quad \eta \in \Gamma_0.$$

*Remark 2.1.* The functions

$$k_\mu^{(n)} : (\mathbb{R}^d)^n \longrightarrow \mathbb{R}_+ \quad (5)$$

$$k_\mu^{(n)}(x_1, \dots, x_n) := \begin{cases} k_\mu(\{x_1, \dots, x_n\}), & \text{if } (x_1, \dots, x_n) \in \widetilde{(\mathbb{R}^d)^n} \\ 0, & \text{otherwise} \end{cases}$$

for  $\sigma$  - Lebesgue measure are well known *correlation functions* in statistical physics, see e.g [30], [31].

For the technical purposes we also recall the following result:

**Lemma 2.1.** *Let  $n \in \mathbb{N}$ ,  $n \geq 2$ , and  $z > 0$  be given. Then*

$$\begin{aligned} \int_{\Gamma_0} \dots \int_{\Gamma_0} G(\eta_1 \cup \dots \cup \eta_n) H(\eta_1, \dots, \eta_n) d\lambda_{z\sigma}(\eta_1) \dots d\lambda_{z\sigma}(\eta_n) = \\ = \int_{\Gamma_0} G(\eta) \sum_{(\eta_1, \dots, \eta_n) \in \mathcal{P}_n(\eta)} H(\eta_1, \dots, \eta_n) d\lambda_{z\sigma}(\eta) \end{aligned}$$

for all measurable functions  $G : \Gamma_0 \mapsto \mathbb{R}$  and  $H : \Gamma_0 \times \dots \times \Gamma_0 \mapsto \mathbb{R}$  with respect to which both sides of the equality make sense. Here  $\mathcal{P}_n(\eta)$  denotes the set of all ordered partitions of  $\eta$  in  $n$  parts, which may be empty.

This lemma is known in the literature as *Minlos lemma* (cf., [21], [26]) and it will be crucial for calculations in many places below.

### 3 Glauber dynamics with competition

We study the special class of Glauber dynamics on  $\Gamma$  with the birth rate equal to a constant (see [16], [17],) and the death rate equal to some unbounded function. In applications this death rate may be considered as the reflection of a competition between particles of the system.

#### 3.1 Potential and Gibbs measures on configuration spaces

A pair potential is a Borel, even function  $\phi : \mathbb{R}^d \mapsto \mathbb{R} \cup \{+\infty\}$ . We assume that  $\phi$  satisfies the following standard conditions, known from statistical physics:

(S) (Stability) There exists  $B > 0$  such that, for any  $\eta \in \Gamma_0$ ,  $|\eta| \geq 2$ .

$$E(\eta) := \sum_{\{x, y\} \subset \eta} \phi(x - y) \geq -B|\eta|.$$

Notice that the stability condition implies that the potential  $\phi$  is semi-bounded from below.

(SI) (Strong Integrability) For any  $\beta > 0$ ,

$$C_{st}(\beta) := \int_{\mathbb{R}^d} |1 - \exp[\beta\phi(x)]| dx < \infty.$$

Throughout the paper we assume that the conditions **(S)** and **(SI)** are satisfied. For  $\gamma \in \Gamma$  and  $x \in \mathbb{R}^d \setminus \gamma$  we define the relative energy of interaction as follows:

$$E(x, \gamma) := \begin{cases} \sum_{y \in \gamma} \phi(x - y), & \text{if } \sum_{y \in \gamma} |\phi(x - y)| < \infty, \\ +\infty, & \text{otherwise.} \end{cases}$$

A probability measure  $\mu$  on  $(\Gamma, \mathcal{B}(\Gamma))$  is called a Gibbs measure with parameters  $\beta > 0$  and  $z > 0$  if it satisfies

$$\int_{\Gamma} \sum_{x \in \gamma} F(\gamma, x) \mu(d\gamma) = \int_{\Gamma} \int_{\mathbb{R}^d} e^{-\beta E(x, \gamma)} F(\gamma \cup x, x) z dx \mu(d\gamma) \quad (6)$$

for any measurable function  $F : \Gamma \times \mathbb{R}^d \rightarrow [0, +\infty]$ . Notice that any fixed set  $\gamma \in \Gamma$  has zero Lebesgue measure, so that the expression  $E(x, \gamma)$  on the right hand side of (6) is almost surely well-defined. The set of all Gibbs measures, which correspond to the potential  $\phi$ , activity parameter  $z > 0$ , and inverse temperature  $\beta > 0$ , will be denoted by  $\mathcal{G}(\phi, z, \beta)$ . For the fixed potential  $\phi$  we will write  $\mathcal{G}(z, \beta)$  instead of  $\mathcal{G}(\phi, z, \beta)$ .

*Remark 3.1.* It is well-known (see e.g. [26], [30]) that if  $\phi$  is stable and

$$C(\beta) := \int_{\mathbb{R}^d} |1 - e^{-\beta \phi(x)}| dx < z^{-1} e^{-1-2B\beta}$$

then the class  $\mathcal{G}(\phi, z, \beta)$  is non-empty. Moreover, under these conditions, for any  $\mu \in \mathcal{G}(\phi, z, \beta)$  the corresponding correlation function  $k_{\mu}$  satisfies the following bound

$$k_{\mu}(\eta) \leq \text{const } C(\beta)^{-|\eta|}, \quad \eta \in \Gamma_0.$$

### 3.2 Generator of Glauber type dynamics

The mechanism of an evolution of configurations in  $\Gamma$  we determine by some formally given generator. The action of such generator in the case of Glauber type dynamics has the following form (see e.g. [16])

$$(LF)(\gamma) := \sum_{x \in \gamma} e^{\beta E(x, \gamma)} D_x^- F(\gamma) + \int_{\mathbb{R}^d} D_x^+ F(\gamma) z dx,$$

where  $D_x^- F(\gamma) = F(\gamma \setminus x) - F(\gamma)$  and  $D_x^+ F(\gamma) = F(\gamma \cup x) - F(\gamma)$ .

*Remark 3.2.* It is known that Gibbs measure  $\mu \in \mathcal{G}(\varkappa, \beta)$  is reversible with respect to the Markov process associated with  $L$  (i.e. the operator  $L$  is symmetrical in  $L^2(\Gamma, \mu)$ ).

*Remark 3.3.* In the case, if  $E(x, \gamma)$  is given via a potential with a non-trivial positive part, the death rate of the operator  $L$  will be unbounded. In the considered model, the death rate reflects a competition between points of the configuration. In the spatial ecology models such a situation is related to the density dependent mortality notion [6].

### 3.3 Spectral properties of the symbol

Let us consider the operator  $L$  on functions  $\mathcal{FL}^0(\Gamma, \mathcal{B}_\Lambda(\Gamma))$ . One can easily check that this operator has the Markov property (it satisfies the maximum principle for the generators of Markov semigroups). Therefore, one may think about this operator as about Markov pre-generator. As it was shown in [16] the following result holds

**Proposition 3.1.** *The image of  $L$  under the  $K$ -transform (or symbol of  $L$ ) on functions  $G \in B_{\text{bs}}(\Gamma_0)$  is given by*

$$\hat{L} := K^{-1}LK = L_0 + L_1 + L_2,$$

where

$$\begin{aligned} L_0G(\eta) &:= -A(\eta)G(\eta), & A(\eta) &= \sum_{x \in \eta} \prod_{y \in \eta \setminus x} e^{\beta\phi(x-y)}; \\ L_1G(\eta) &:= - \sum_{\xi \subset \eta, \xi \neq \eta} G(\xi) \sum_{x \in \xi} \prod_{y \in \xi \setminus x} e^{\beta\phi(x-y)} \prod_{y \in \eta \setminus \xi} (e^{\beta\phi(x-y)} - 1); \\ L_2G(\eta) &:= \varkappa \int_{\mathbb{R}^d} G(\eta \cup x) dx, & \varkappa &> 0. \end{aligned}$$

We study the operator  $\hat{L}$  in the Banach space

$$\mathcal{L}_C := L^1\left(\Gamma_0, C^{|\eta|} \lambda(d\eta)\right),$$

where  $C > 0$  and  $\lambda := \lambda_1$  is the Lebesgue-Poisson measure with intensity 1. To define the symbol of  $L$  mathematically rigorously we introduce its domain (see e.g. [16])

$$D(\hat{L}) := \{G \in \mathcal{L}_C : A \cdot G \in \mathcal{L}_C\}.$$

*Remark 3.4.* In our recent paper [16] we have shown that for any triple of positive constants  $C$ ,  $\varkappa$  and  $\beta$  which satisfies

$$2e^{Cst(\beta)C} + 2\varkappa e^{2B\beta}C^{-1} < 3$$

the symbol  $\hat{L}$  is a generator of a holomorphic semigroup  $T_t$  in  $\mathcal{L}_C$ . Moreover, there exists a non-equilibrium Markov process which corresponds to  $L$ .

Now, we set

$$L_0^1 := \{G \in \mathcal{L}_C \mid G = \text{const } \psi_0\}, \quad \psi_0(\eta) = \begin{cases} 1, & \eta = \emptyset, \\ 0, & \eta \neq \emptyset \end{cases} \quad (7)$$

and

$$\mathcal{L}_{\geq 1} := \{G \in \mathcal{L}_C : G(\emptyset) = 0\}.$$

Then, the space  $\mathcal{L}_C$  can be decomposed as

$$\mathcal{L}_C = L_0^1 + \mathcal{L}_{\geq 1}.$$

One should note that the decomposition of any  $G \in \mathcal{L}_C$  into the sum of functions from  $L_0^1$  and  $\mathcal{L}_{\geq 1}$  is unique. As result the operator  $\hat{L}$  can be represented in the form

$$\hat{L} := \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix},$$

where  $L_{00} = 0$ ,  $L_{10} = 0$ ,

$$L_{01}G(\emptyset) = \varkappa \int_{\mathbb{R}^d} G^{(1)}(x)dx, \quad G = (G^{(n)}, n \geq 0) \in \mathcal{L}_{\geq 1}.$$

Next we introduce the Banach space

$$\mathcal{K}_C := \left\{ k : \Gamma_0 \rightarrow \mathbb{R} \mid k \cdot C^{-|\cdot|} \in L^\infty(\Gamma_0, \lambda) \right\},$$

which is dual to  $\mathcal{L}_C$  with respect to the duality defined as

$$\ll G, k \gg := \int_{\Gamma_0} G(\eta) k(\eta) \lambda(d\eta). \quad (8)$$

Analogously, to the decomposition of the space  $\mathcal{L}_C$  we get the following representation for the space  $\mathcal{K}_C$ :

$$\mathcal{K}_C = L_0^\infty + \mathcal{K}_{\geq 1},$$

where

$$L_0^\infty := \{k \in \mathcal{L}_C \mid k = \text{const } \psi_0\}, \quad (9)$$

and

$$\mathcal{K}_{\geq 1} := \{k \in \mathcal{K}_C : k(\emptyset) = 0\}.$$

The corresponding adjoint operator to the operator  $L$  on  $\mathcal{K}_C$  we denote by

$$\hat{L}^* := \begin{pmatrix} L_{00}^* & L_{01}^* \\ L_{10}^* & L_{11}^* \end{pmatrix},$$

where  $L_{00}^* = 0$ ,  $L_{01}^* = 0$ ,

The spectral properties of the operator  $\hat{L}$  are described in the following theorem

**Theorem 3.1.**

1. The point  $z = 0$  is an eigenvalue of the operator  $\hat{L}$  with the eigenvector  $\psi_0$  (see (7)).
2. There exists  $z_0 > 0$  such that

$$I_1 = \{z \in \mathbb{C} : \text{Re} z > -z_0\} \setminus \{0\}$$

and

$$I_2 = \left\{ z \in \mathbb{C} : |\arg z| < \frac{3\pi}{4} \right\} \setminus \{0\}$$

belong to the resolvent set of the operator  $\hat{L}$ .

*Proof.* We will show that for any  $u \geq 0$  and  $w \in \mathbb{R}$  the operator  $(L_{11} - (u + iw)\mathbb{1})^{-1}$  in the space  $\mathcal{L}_{\geq 1}$  exists and bounded. Indeed,

$$(L_{11} - (u + iw)\mathbb{1})^{-1} =$$

$$(\mathbb{1} + (L_0^{11} - (u + iw)\mathbb{1})^{-1}(L_1^{11} + L_2^{11}))^{-1}(L_0^{11} - (u + iw)\mathbb{1})^{-1},$$

where  $L_i^{11}$  - the part of the operator  $L_i$  in the space  $\mathcal{L}_{\geq 1}$ ,  $i = 0, 1, 2$ . The resolvent  $(L_0^{11} - (u + iw)\mathbb{1})^{-1}$  exists in the space  $\mathcal{L}_{\geq 1}$  since

$$A(\eta) > 0, \quad \forall \eta \neq \emptyset.$$

In order to claim that the resolvent

$$(\mathbb{1} + (L_0^{11} - (u + iw)\mathbb{1})^{-1}(L_1^{11} + L_2^{11}))^{-1}$$

exists it is enough to show that

$$\|(L_0^{11} - (u + iw)\mathbb{1})^{-1}(L_1^{11} + L_2^{11})\| \leq 1.$$

It is easy to see that for  $\eta \neq \emptyset$

$$(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_1^{11}G(\eta) = \frac{\sum_{\xi \subset \eta, \xi \neq \eta} G(\xi) \sum_{x \in \xi} \prod_{y \in \xi \setminus x} e^{\beta\phi(x-y)} \prod_{y \in \eta \setminus \xi} (e^{\beta\phi(x-y)} - 1)}{\sum_{x \in \eta} \prod_{y \in \eta \setminus x} e^{\beta\phi(x-y)} + u + iw}.$$

As result

$$|(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_1^{11}G(\eta)| < A^{-1}(\eta) \left[ \sum_{\xi \subset \eta} |G(\xi)| \sum_{x \in \xi} e^{\beta E(x, \xi \setminus x)} B(\eta \setminus \xi, x) - |G(\eta)| A(\eta) \right], \quad \eta \neq \emptyset$$

where  $B(\eta, x) := \prod_{y \in \eta} |e^{\beta\phi(x-y)} - 1|$ . Then, using Minlos lemma

$$\begin{aligned} & \int_{\Gamma_0 \setminus \{\emptyset\}} |(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_1^{11}G(\eta)| C^{|\eta|} \lambda(d\eta) \leq \\ & \int_{\Gamma_0 \setminus \{\emptyset\}} \lambda(d\xi_1) \int_{\Gamma_0} \lambda(d\xi_2) \frac{|G(\xi_1)| \sum_{x \in \xi_1} e^{\beta E(x, \xi_1 \setminus x)} B(\xi_2, x) C^{|\xi_1|} C^{|\xi_2|}}{\sum_{x \in \xi_1} e^{\beta E(x, \xi_1 \setminus x)} e^{-\beta m |\xi_2|}} - \\ & - \int_{\Gamma_0 \setminus \{\emptyset\}} \frac{|G(\eta)| \sum_{x \in \eta} \prod_{y \in \eta \setminus x} e^{\beta\phi(x-y)}}{\sum_{x \in \eta} \prod_{y \in \eta \setminus x} e^{\beta\phi(x-y)}} \lambda(d\eta). \end{aligned} \quad (10)$$

where  $m := -\min\{0, \min \phi\}$ . For the last bound we have used the following inequality:

$$\begin{aligned} & \sum_{x \in \xi_1 \cup \xi_2} \prod_{y \in \xi_1 \cup \xi_2 \setminus \{x\}} e^{\beta\phi(x-y)} > \\ & \sum_{x \in \xi_1} \prod_{y \in \xi_1 \setminus x} e^{\beta\phi(x-y)} \prod_{y \in \xi_2} e^{\beta\phi(x-y)} > \sum_{x \in \xi_1} \prod_{y \in \xi_1 \setminus x} e^{\beta\phi(x-y)} e^{-\beta m |\xi_2|}. \end{aligned}$$

The estimate (10) gives

$$\begin{aligned} & \int_{\Gamma_0 \setminus \{\emptyset\}} |(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_1^{11}G(\eta)| C^{|\eta|} \lambda(d\eta) < \\ & < \|G\| \exp \left\{ e^{\beta m} C C_{st}(\beta) \right\} - \|G\| = \left( \exp \left\{ e^{\beta m} C C_{st}(\beta) \right\} - 1 \right) \|G\|. \end{aligned}$$

It is clear that for a small  $\beta$

$$\exp \left\{ e^{\beta m} C C_{st}(\beta) \right\} - 1 < \frac{1}{3}.$$

and hence also

$$\|(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_1^{11}\| < \frac{1}{3}.$$

Next we estimate the norm of  $(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_2^{11}$ . The simple inequality

$$|(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_2^{11}G(\eta)| \leq \varkappa A^{-1}(\eta) \int_{\mathbb{R}^d} G(\eta \cup x) dx, \quad \eta \neq \emptyset$$

yields

$$\begin{aligned} & \int_{\Gamma_0 \setminus \{\emptyset\}} |(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_2^{11}G(\eta)| C^{|\eta|} \lambda(d\eta) < \\ & < \varkappa e^{2B\beta} \int_{\Gamma_0 \setminus \{\emptyset\}} \frac{1}{|\eta|} \int_{\mathbb{R}^d} |G(\eta \cup x)| C^{|\eta|} dx \lambda(d\eta) = \\ & = \varkappa e^{2B\beta} \int_{\Gamma_0 \setminus (\Gamma_0^{(1)} \cup \{\emptyset\})} \frac{|\eta|}{|\eta| - 1} C^{|\eta| - 1} |G(\eta)| \lambda(d\eta) \leq \frac{2\varkappa e^{2B\beta}}{C} \|G\|, \end{aligned}$$

where  $B > 0$  is a constant which is determined from the stability condition for  $\phi$ . In the estimate above we have also used the following inequality

$$\frac{1}{|\eta|} \sum_{x \in \eta} \prod_{y \in \eta \setminus x} e^{\beta\phi(x-y)} \geq \exp \left\{ \frac{2\beta}{|\eta|} E(\eta) \right\} \geq e^{-2\beta B}.$$

Therefore, for small  $\varkappa$  we have

$$\|(L_0^{11} - (u + iw)\mathbb{1})^{-1}L_2^{11}\| < \frac{1}{3}$$

and finally

$$\|(L_0^{11} - (u + iw)\mathbb{1})^{-1}(L_1^{11} + L_2^{11})\| < \frac{2}{3}.$$

The latter fact implies

$$\begin{aligned} & \|(L_{11} - (u + iw)\mathbb{1})^{-1}\| = \\ & = \|(\mathbb{1} + (L_0^{11} - (u + iw)\mathbb{1})^{-1}(L_1^{11} + L_2^{11}))^{-1}(L_0^{11} - (u + iw)\mathbb{1})^{-1}\| < \\ & < 3\|(L_0^{11} - (u + iw)\mathbb{1})^{-1}\|, \end{aligned}$$

but

$$\begin{aligned} \|(L_0^{11} - (u + iw)\mathbb{1})^{-1}\| &\leq \frac{1}{\left(\left(\min_{|\xi| \geq 1} A(\xi)\right)^2 + w^2\right)^{\frac{1}{2}}} \leq \\ &\leq \frac{1}{\left(\left(\min_{|\xi| \geq 1} |\xi| e^{-2\beta B}\right)^2 + w^2\right)^{\frac{1}{2}}} = \frac{1}{(e^{-4\beta B} + w^2)^{\frac{1}{2}}} \end{aligned}$$

Therefore, since

$$\|(L_{11} - (u + iw)\mathbb{1})^{-1}\| = \|(\mathbb{1} - (L_{11} - iw\mathbb{1})^{-1}u)^{-1} (L_{11} - iw\mathbb{1})^{-1}\|$$

for  $0 \geq u \geq -z_0$ ,  $z_0 = \frac{1}{3}e^{-2\beta B}$  we have

$$\|(L_{11} - (u + iw)\mathbb{1})^{-1}\| \leq 3 \left(1 - \frac{|u|}{z_0}\right)^{-1} \frac{1}{(e^{-4\beta B} + w^2)^{\frac{1}{2}}}$$

As result, for all  $z \in I_1$  the operator  $(L_{11} - z\mathbb{1})^{-1}$  exists and bounded.

Now let us note that  $|w| > |u|$  for all  $z = u + iw \in I_2$ ,  $u < 0$ . As result

$$|A(\eta) + u + iw| > ((A(\eta) + u)^2 + w^2)^{\frac{1}{2}} \geq \frac{A(\eta)}{\sqrt{2}}.$$

Using the same arguments as before one can show now that  $I_2$  belongs to the resolvent set of  $L_{11}$  as well.

The spectral properties of the operator  $L_{11}$ , studied above, play the key role for the proof of the main statements of the theorem. Below we prove the facts claimed in the theorem.

It is clear that vector  $\psi_0$  is the eigenvector of the operator  $\hat{L}$  with the corresponding eigenvalue 0. Suppose that there exists another eigenvector  $\bar{\psi} = (\bar{\psi}^0, \bar{\psi}^1) \in \mathcal{L}_C$ ,  $\bar{\psi}^0 \in L_0^1$ ,  $\bar{\psi}^1 \in \mathcal{L}_{\geq 1}$  which corresponds to the eigenvalue 0. Then the following should be true

$$L_{00}\bar{\psi}^0 + L_{01}\bar{\psi}^1 = 0$$

$$L_{11}\bar{\psi}^1 = 0.$$

The last equality implies that  $\bar{\psi}^1 = 0$ , and hence  $\bar{\psi} = \text{const} \bar{\psi}_0$  that proves the first statement of the theorem. To prove the second one, we study for any  $z \in I_1 \cup I_2$ ,  $z \neq 0$  the following equation

$$(\hat{L} - z\mathbb{1})R = G, \quad R, G \in \mathcal{L}_C.$$

It can be rewritten also in the form

$$(L_{00} - z\mathbb{1})R^0 + L_{01}R^1 = G^0$$

$$(L_{11} - z\mathbb{1})R^1 = G^1,$$

where  $R = (R^0, R^1)$ ,  $G = (G^0, G^1)$ , and  $R^0, G^0 \in \mathcal{L}_0^1$ ,  $R^1, G^1 \in \mathcal{L}_{\geq 1}$ . The solution to the second equation exists and unique:

$$R^1 = (L_{11} - z\mathbb{1})^{-1}G^1.$$

Therefore,

$$R^0 = \frac{1}{z} (L_{01}(L_{11} - z\mathbb{1})^{-1}G^1 - G^0)$$

As result, the resolvent for the operator  $\hat{L}$  is defined for all  $z \in I_1 \cup I_2$ ,  $z \neq 0$ .  $\square$

Since the operator  $\hat{L}$  in  $\mathcal{L}_C$  is closed and densely defined, the corresponding adjoint operator  $\hat{L}^*$  in  $\mathcal{K}_C$  with respect to the duality (8) has the same spectrum as the operator  $\hat{L}$  (see e.g. [12]). In particular, the subspace

$$\mathcal{K}_{inv}^* := \{k \in \mathcal{K}_C \mid \ll \psi_0, k \gg = 0\} \subset \mathcal{K}_C$$

is invariant for the operator  $\hat{L}^*$ . The latter subspace can be identified with the subspace  $\mathcal{K}_{\geq 1}$ . The restriction of the operator  $\hat{L}^*$  to  $\mathcal{K}_{\geq 1}$  coincides with the operator  $L_{11}^*$ .

## 4 Ergodicity

The space  $\mathcal{L}_0^1$  is invariant with respect to the semigroup  $T_t$ . As result, there exists the factor semigroup on the factor space  $\mathcal{L}_C / \mathcal{L}_0^1$ , which can be identified with the space  $\mathcal{L}_{\geq 1}$ , and the operator  $\hat{L} / \mathcal{L}_0^1$  in this space, the action of which is given as

$$\left( \hat{L} / \mathcal{L}_0^1 \right) [G] := [LG], \quad (11)$$

where  $[G] \in \mathcal{L}_C / \mathcal{L}_0^1$  is the equivalent class of the element  $G \in \mathcal{L}_C$ . Since  $\mathcal{L}_0^1 \subset \mathcal{L}_C$  is an invariant subspace for  $\hat{L}$ , the definition (11) is correct. Identifying the space  $\mathcal{L}_C / \mathcal{L}_0^1$  with  $\mathcal{L}_{\geq 1}$  one can see that the operator

$\hat{L}/L_0^1$  corresponds to the operator  $L_{11}$  in the sense of similarity, i.e. there exists an (isometric) isomorphism  $J$  such that

$$J^{-1}L_{11}J = \hat{L}/L_0^1.$$

The same transformation can be shifted also to the corresponding semigroups, i.e.

$$\tilde{T}_t := J \left( T_t / L_0^1 \right) J^{-1},$$

which shows the existence of the semigroup  $\tilde{T}_t$ . Moreover, it is clear by the construction that its generator coincides with  $L_{11}$ .

Let us consider the adjoint evolution  $T_t^*$  to the evolution  $T_t$  with respect to the duality (8) and its restriction  $\tilde{T}_t^* = T_t^*|_{\mathcal{K}_{\geq 1}}$  on the invariant subspace  $\mathcal{K}_{\geq 1}$ . It is easy to see that  $T_t^*$  has semigroup property and, moreover,  $\tilde{T}_t^*$  is the adjoint semigroup to the semigroup  $\tilde{T}_t$  with respect to the duality (8).

The following theorem is crucial for the study of the ergodicity of the process.

**Theorem 4.1.** *Suppose that the parameters of the system satisfy the following conditions*

$$\exp \{ e^{2\beta B} C_{st}(\beta) C \} + 2\kappa e^{2\beta B} C^{-1} < \frac{3}{2}$$

then for  $t > 1$

$$\| \tilde{T}_t^* \| \leq \text{const} e^{-\frac{z_0}{2}t}.$$

*Proof.* Using the inverse formula (see [9]) we have

$$t\tilde{T}_t = -\frac{1}{2\pi i} \int_{\Upsilon} (L_{11} - z\mathbb{1})^{-2} e^{zt} dz,$$

where integral is taken over the contour  $\Upsilon$  of the form

$$\Upsilon = \{ z = -z_0/2 + iw : w \in \mathbb{R} \}.$$

provided, of course, that the right-hand side of the formula exists. Indeed,

$$\begin{aligned} & \left\| \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\frac{z_0}{2}t} e^{iwt} (L_{11} - (-\frac{z_0}{2} + iw)\mathbb{1})^{-2} dw \right\| < \\ & < \frac{1}{2\pi} e^{-\frac{z_0}{2}t} \int_{-\infty}^{\infty} \frac{9}{e^{-4\beta B} + w^2} dw = \text{const} e^{-\frac{z_0}{2}t} \end{aligned}$$

Finally, for  $t > 1$

$$\|\tilde{T}_t^*\| \leq \|t\tilde{T}_t\| < \text{const } e^{-\frac{z_0}{2}t}, \quad (12)$$

where we used the fact that  $\|\tilde{T}_t\| = \|\tilde{T}_t^*\|$  □

Now, given an initial correlation function  $k_0$  from the class  $\mathcal{K}_C$ , we look to the corresponding dynamics  $k_t := T_t^* k_0$ . We would like to show that  $k_t$  converges to the invariant correlation function as  $t \rightarrow 0$ . Toward this end we should adjust the initial correlation functions with the possible parameters of our system. The corresponding result is described below in the theorem.

**Theorem 4.2.** *Let the initial measure  $\mu_0 \in \mathcal{M}^1(\Gamma)$  has the correlation function  $k_0 \in \mathcal{K}_C$  for some  $C > 0$ . Suppose that the parameters of the system satisfy the following conditions*

1.

$$\exp\{e^{2\beta B} C_{st}(\beta)C\} + 2\kappa e^{2\beta B} C^{-1} < \frac{3}{2};$$

2.

$$\kappa e^{2\beta B} C^{-1} \exp\{e^{2\beta B} C C_{st}(\beta)\} < 1,$$

where  $B$  is the same as in the stability condition. Denote  $k_\mu$  the correlation function corresponding to  $\mu \in \mathcal{G}(\kappa, \beta)$ . Then,

$$\|k_t - k_\mu\|_{\mathcal{K}_C} \leq \text{const } e^{-\frac{z_0}{2}t} \|k_0 - k_\mu\|_{\mathcal{K}_C}.$$

*Proof.* Let us clarify the sense of conditions of the theorem. The condition 1 ensures the existence of the semigroup  $T_t^* := e^{t\hat{L}^*}$  in the Banach space  $\mathcal{K}_C$ . Moreover

$$\|\tilde{T}_t^*\| \leq \text{const } e^{-\frac{z_0}{2}t}.$$

The condition 2 gives the existence of  $\mu \in \mathcal{G}(\kappa, \beta)$  and the following bound for the corresponding correlation function

$$k_\mu(\eta) \leq C^{|\eta|}, \quad \eta \in \Gamma_0. \quad (13)$$

Since  $\mu$  is invariant measure for the operator  $L$  it is not difficult to see using the properties of  $K$ -transform that

$$e^{t\hat{L}^*} k_\mu = k_\mu.$$

Denote

$$r_t := k_t - k_\mu,$$

where  $k_t := e^{t\hat{L}^*} k_0$  and  $k_0 \in \mathcal{K}_C$  is a correlation function of some measure  $\mu_0 \in \mathcal{M}^1(\Gamma)$ . Since  $k_0(\emptyset) = 1$  and  $k_\mu(\emptyset) = 1$ , we have  $r_0(\emptyset) = 0$ . The condition 2 implies that  $r_0$  is a function from  $\mathcal{K}_{\geq 1}$ . Indeed,

$$|r_0| = |k_0 - k_\mu| < \text{const } C^{|\eta|}.$$

Finally, using Theorem 4.1 we have

$$\|k_t - k_\mu\|_{\mathcal{K}_C} = \|\tilde{T}_t^* r_0\|_{\mathcal{K}_C} \leq \text{const } e^{-\frac{z_0}{2}t} \|k_0 - k_\mu\|_{\mathcal{K}_C},$$

that concludes the proof of the theorem.  $\square$

*Remark 4.1.* For any fixed  $C > 0$  there exist small  $\varkappa > 0$  and  $\beta > 0$  such that the conditions of Theorem 4.2 are fulfilled.

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## References

- [1] S. Albeverio, Yu. Kondratiev, M. Röckner, Analysis and geometry on configuration spaces, *J. Func. Anal.* **154**, (1998) 444-500.
- [2] Yu. M. Berezansky, Yu. G. Kondratiev, T. Kuna, E. Lytvynov, On a spectral representation for correlation measures in configuration space analysis, *Methods Funct. Anal. Topology* **5**, No.4 (1999) 87-100.
- [3] L. Bertini, N. Cancrini, F. Cesi, The spectral gap for a Glauber-type dynamics in a continuous gas, *Ann. Inst. H. Poincaré Probab. Statist.* **38** (2002) 91–108.
- [4] R. L. Dobrushin, Ya. G. Sinai, Yu. M. Suhov, *Dynamical System of the Statistical Mechanics, Itogi Nauki i Tekhniki, VINITI Akad. Nauk SSSR, ser. Sovremennyye Problemy Matematiki, Fundamental'nye napravleniya* **2** (1985) 235-284.
- [5] R. Fernandez, P. Ferrari and G. Guerberoff, Spatial birth-and-death processes in random environment, *Math. Phys. Electron. J.* **11** (2005) Paper 3, 52pp.
- [6] D. L. Finkelshtein, Yu. G. Kondratiev, O. Kutovyi, Individual based model with competition in spatial ecology, <http://arxiv.org/abs/0803.3565>. Submitted to: *SIAM Journal on Mathematical Analysis*.
- [7] N. Garcia and T. Kurtz, Spatial birth and death processes as solutions of stochastic equations, *ALEA Lat. Am. J. Probab. Math. Stat.* **1** (2006) 281–303.
- [8] H. O. Georgii, **Gibbs measures and Phase Transitions**, (Walter de Gruyter, 1982).
- [9] E. Hille and R. Phillips, **Functional analysis and semi-groups** American Mathematical Society Colloquium Publications, vol. 31. American Mathematical Society, Providence, R. I., 1957 .
- [10] R. A. Holley, D. W. Stroock, Nearest neighbor birth and death processes on the real line, *Acta Math.* **140** (1978) 103–154.

- [11] K. Ito, F. Kappel **Evolution equations and approximations** (New Jersey, London, Singapore, Hong Kong: World Scientific, 2002)
- [12] T. Kato, **Perturbation theory for linear operators** (Berlin, Heidelberg, New York: Springer Verlag, 1966)
- [13] Yu. G. Kondratiev and T. Kuna, Harmonic analysis on configuration space I. General theory, *Infinite Dimensional Analysis, Quantum Probability and Related Topics* **5**, no.2 (2002) 201-233.
- [14] Yu. G. Kondratiev, T. Kuna, M. J. Oliveira, J. L. da Silva and L. Streit, Hydrodynamic limits for the free Kawasaki dynamics of continuous particle systems *SFB-701 Preprint, University of Bielefeld, Bielefeld, Germany*(2007).
- [15] Yu. G. Kondratiev, O. V. Kutoviy, On the metrical properties of the configuration space *Math. Nachr.* **279**, No.7, (2006) 774–783.
- [16] Yu. G. Kondratiev, O. V. Kutoviy, R. A. Minlos On non-equilibrium stochastic dynamics for interacting particle systems in continuum *J. Funct. Anal.* **255**, No.7, (2007) 200–227.
- [17] Yu. G. Kondratiev, O. V. Kutoviy, E. Zhizhina, Nonequilibrium Glauber-type dynamics in continuum. *J. Math. Phys.* **47**, No.11 (2006) 17pp.
- [18] Yu. G. Kondratiev and E. Lytvynov, Glauber dynamics of continuous particle systems *Ann. Inst. H.Poincaré Probab. Statist.* **41**, No.4, (2005) 685–702.
- [19] Yu. G. Kondratiev, E. Lytvynov, M. Röckner, Equilibrium Kawasaki dynamics of continuous particle systems. *Infin. Dimens. Anal. Quantum Probab. Relat. Top.*, **10**, No.2, (2007) 185–209.
- [20] Yu. G. Kondratiev, E. Lytvynov, M. Röckner, Infinite interacting diffusion particles. I. Equilibrium process and its scaling limit. *Forum Math.* **18**, No.1, (2006) 9–43.
- [21] Yu. G. Kondratiev, R. Minlos, and E. Zhizhina, One-particle subspaces of the generator of Glauber dynamics of continuous particle systems. *Rev. Math. Phys.*, **16**, No.9, (2004) 1-42.

- [22] Yu. G. Kondratiev and A. Skorokhod, On contact process in continuum *Infin. Dimens. Anal. Quantum Probab. Relat. Top.*, **9**, No.2, (2006) 187-198.
- [23] A. Lenard, States of classical statistical mechanical systems of infinitely many particles. I, *Arch. Rational Mech. Anal.* **59** (1975) 219-239.
- [24] A. Lenard, States of classical statistical mechanical systems of infinitely many particles. II, *Arch. Rational Mech. Anal.* **59** (1975) 241-256.
- [25] T. M. Liggett, **Interacting Particle Systems**, Springer-Verlag, 1985.
- [26] R. A. Minlos, **Lectures on statistical physics**, *UMN* (Russian) No.1, (1968) 133-190.
- [27] C. Preston, **Random Fields**, *Lecture notes in Mathematics, Vol.534*, (Berlin Heidelberg, New York: Springer, 1976).
- [28] C. Preston, Spatial birth-and-death processes, *in Proceedings of the 40th Session of the International Statistical Institute (Warsaw, 1975)* **2**, *Bull. Inst. Internat. Statist.* **46** (1975) 371-391.
- [29] M. Reed, B. Simon, **Methods of modern mathematical physics, 2. Fourier Analysis, Self-Adjointness**, (New York, London: Academic Press, 1972).
- [30] D. Ruelle, **Statistical Mechanics** (New York, Benjamin, 1969).
- [31] D. Ruelle, Superstable interactions in classical statistical mechanics, *Commun. Math. Phys.* **18** (1970) 127-159.
- [32] L. Wu, Estimate of spectral gap for continuous gas, *Ann. Inst. H. Poincaré Probab. Statist.* **40** No.4 (2004) 387-409.