

# AN EILENBERG–Ganea PHENOMENON FOR ACTIONS WITH VIRTUALLY CYCLIC STABILISERS

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ABSTRACT. In dimension 3 and above, Bredon cohomology gives an accurate purely algebraic description of the minimal dimension of the classifying space for actions of a group with stabilisers in any given family of subgroups. For some Coxeter groups and the family of virtually cyclic subgroups we show that the Bredon cohomological dimension is 2 while the Bredon geometric dimension is 3.

## 1. INTRODUCTION AND PRELIMINARIES

For a discrete group  $G$ , a family of subgroups  $\mathfrak{F}$  is a non-empty collection of subgroups of  $G$  that is closed under conjugation and taking subgroups. If  $\mathfrak{F}$  is a family of subgroups of  $G$  then a model for  $E_{\mathfrak{F}}G$ , the classifying space for  $G$ -actions with stabilisers in  $\mathfrak{F}$ , is a  $G$ -CW-complex  $X$  such that for  $H \leq G$ , the fixed point set  $X^H$  is empty if  $H \notin \mathfrak{F}$  and is contractible if  $H \in \mathfrak{F}$ . For any  $G$  and  $\mathfrak{F}$  there is always a model for  $E_{\mathfrak{F}}G$  and it is unique up to equivariant homotopy.

In the case when  $\mathfrak{F}$  consists of just the trivial group,  $E_{\mathfrak{F}}G$  is the same thing as  $EG$ , the universal cover of an Eilenberg–Mac Lane space for  $G$ . In the case when  $\mathfrak{F}$  is the family  $\mathfrak{F}_{\text{fin}}(G)$  of all finite subgroups of  $G$  (respectively the family  $\mathfrak{F}_{\text{vc}}(G)$  of all virtually cyclic subgroups of  $G$ ) we write  $\underline{E}G$  (respectively  $\underline{\underline{E}}G$ ) for  $E_{\mathfrak{F}}G$ . The minimal dimension of any model for  $E_{\mathfrak{F}}G$  is denoted by  $\text{gd}_{\mathfrak{F}}G$  and is called the *Bredon geometric dimension* of  $G$ .

Homological algebra over the group ring  $\mathbb{Z}G$  can be used to study models for  $EG$ , and Bredon cohomology is the natural generalisation for studying models for  $E_{\mathfrak{F}}G$ . In Bredon cohomology the orbit category  $\mathcal{O}_{\mathfrak{F}}G$  replaces the group  $G$ . The orbit category  $\mathcal{O}_{\mathfrak{F}}G$  is the category with objects the  $G$ -sets  $G/H$  with  $H \in \mathfrak{F}$  and  $G$  maps as morphisms. A (right)  $\mathcal{O}_{\mathfrak{F}}G$ -module is then a contravariant functor from the orbit category  $\mathcal{O}_{\mathfrak{F}}G$  to the category of abelian groups. In the case when  $\mathfrak{F}$  consists of just the trivial group,  $\mathcal{O}_{\mathfrak{F}}G$  is a category with one object and morphism set  $G$  and  $\mathcal{O}_{\mathfrak{F}}G$ -modules are the same as  $\mathbb{Z}G$ -modules.

The category of  $\mathcal{O}_{\mathfrak{F}}G$ -modules is an abelian category with enough projectives. The *Bredon cohomological dimension*  $\text{cd}_{\mathfrak{F}}G$  is defined to be the projective dimension of the trivial  $\mathcal{O}_{\mathfrak{F}}G$ -module  $\mathbb{Z}$ , which takes the value  $\mathbb{Z}$  on any object of  $\mathcal{O}_{\mathfrak{F}}G$  and which maps any morphism to the identity. The derived functors of the morphism functor in the category of Bredon modules over  $\mathcal{O}_{\mathfrak{F}}G$  are denoted by  $\text{Ext}_{\mathfrak{F}}^*(-, -)$ . The *Bredon cohomology groups* of  $G$  with coefficients the  $\mathcal{O}_{\mathfrak{F}}G$ -module  $M$  are the abelian groups  $H_{\mathfrak{F}}^*(G; M) = \text{Ext}_{\mathfrak{F}}^*(\mathbb{Z}; M)$ . For details on Bredon cohomology we refer to [12] or [9].

If the family  $\mathfrak{F}$  consists of the trivial subgroup only, then  $\text{gd}_{\mathfrak{F}} G$  is the minimal dimension  $\text{gd} G$  an Eilenberg–Mac Lane space for  $G$  can have. If  $\mathfrak{F}$  is the family  $\mathfrak{F}_{\text{fin}}(G)$  (respectively  $\mathfrak{F}_{\text{vc}}(G)$ ) then we use the notation  $\underline{\text{gd}} G$  (respectively  $\underline{\underline{\text{gd}}} G$ ) for  $\text{gd}_{\mathfrak{F}} G$ .

As in the classical case a model for  $E_{\mathfrak{F}} G$  gives rise to a resolution of the trivial  $\mathcal{O}_{\mathfrak{F}} G$ -module  $\mathbb{Z}$  by projective  $\mathcal{O}_{\mathfrak{F}} G$ -modules. Therefore  $\text{cd}_{\mathfrak{F}} G \leq \text{gd}_{\mathfrak{F}} G$  in general. If  $\text{cd}_{\mathfrak{F}} G \geq 3$ , then  $\text{cd}_{\mathfrak{F}} G = \text{gd}_{\mathfrak{F}} G$ . In the classical case, that is when  $\mathfrak{F} = \{1\}$  consists only of the trivial subgroup, this is due to Eilenberg–Ganea [7]. For  $\mathfrak{F} = \mathfrak{F}_{\text{fin}}(G)$  this was proved in [12] and this proof generalises to arbitrary families  $\mathfrak{F}$ , cf. Theorem 0.1 in [13, p. 294]. In the classical case it is well known that  $\text{cd} G = 0$  implies  $\text{gd} G = 1$  and for general families this implication follows from Lemma 2.5 in [16, p. 265].

In the classical case, the statement that the cohomological and geometric dimension always agree is known as the Eilenberg–Ganea Conjecture. Since the work of Stallings [14] and Swan [15] implies that  $\text{cd} G = 1$  if and only if  $\text{gd} G = 1$ , this conjecture can only be falsified by a group  $G$  with  $\text{cd} G = 2$  but  $\text{gd} G = 3$ .

In [1] right-angled Coxeter groups  $W$  such that  $\underline{\text{cd}} W = 2$  but  $\underline{\underline{\text{gd}}} W = 3$  were exhibited. We show here that some, but not all, of these examples have a similar property for actions with virtually cyclic stabilisers.

**Main Theorem.** *Let  $(W, S)$  be a right-angled Coxeter system for which the nerve  $L = L(W, S)$  is an acyclic 2-complex that cannot be embedded in any contractible 2-complex.*

- *If  $W$  is word hyperbolic, then*

$$\underline{\text{cd}} W = 2 \quad \text{and} \quad \underline{\underline{\text{gd}}} W = 3.$$

- *If  $W$  is not word hyperbolic, then*

$$\underline{\text{cd}} W = \underline{\underline{\text{gd}}} W \geq 3.$$

A right angled Coxeter group  $W$  is word hyperbolic if and only if its nerve  $L$  satisfies the so called “flag no squares condition”, cf. [4, p. 233]. By Proposition 2.1 of [5] the “flag no squares conditions” puts no restriction on the homeomorphism type of the 2-complex  $L$  (or see [1, p. 498] an explicit example for a suitably triangulated  $L$ ). Therefore it follows from our theorem, that the Bredon analogue of the Eilenberg–Ganea Conjecture is false for the family of virtually cyclic subgroups.

The proof of the non-word hyperbolic case of our Main Theorem is the easy part and is described in Section 3. The word hyperbolic case is Theorem 6 and 7 combined.

As mentioned before, in the classical case  $\text{cd} G = 1$  implies  $\text{gd} G = 1$  by the work of Stallings and Swan. It follows from Dunwoody’s Accessibility Theorem [6], that the same implication is true in the case that  $\mathfrak{F} = \mathfrak{F}_{\text{fin}}(G)$ . In the light of this one may ask, whether this implication also holds in the case that  $\mathfrak{F} = \mathfrak{F}_{\text{vc}}(G)$ . The first author obtained in his thesis a positive answer for countable, torsion-free, soluble groups [9, p. 127]. In this class, the groups  $G$  with  $\underline{\text{cd}} G = 1$  are precisely the subgroups of the rational numbers which are not finitely generated and for these group  $\underline{\underline{\text{gd}}} G = 1$  holds. However, a general answer to this question is still open.

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## 2. COXETER GROUPS AND THE DAVIS COMPLEX

A *Coxeter matrix* is a symmetric matrix  $M = (m_{st})$  indexed by a finite set  $S$  and with entries integers or  $\infty$  subject to the conditions that for all  $s, t \in S$

- (1)  $m_{st} = 1$  if  $s = t$ , and
- (2)  $m_{st} \geq 2$  otherwise.

Associated to a Coxeter matrix  $M$  one has the *Coxeter group*  $W$  given by the presentation

$$W = \langle S \mid (st)^{m_{st}} = 1 \text{ for all } s, t \in S \text{ with } m_{st} \neq \infty \rangle.$$

The Coxeter group  $W$  is *right-angled* if the finite off-diagonal entries of the Coxeter matrix are all equal to 2. The elements of  $S$  are called the *fundamental Coxeter generators* of the Coxeter group  $W$  and the pair  $(W, S)$  is called a *Coxeter system*. If  $T \subset S$ , then  $W_T$  denotes the subgroup of  $W$  generated by  $T$  and these subgroups are called *special*.

The *nerve*  $L = L(W, S)$  of a Coxeter system  $(W, S)$  is the simplicial complex with vertex set  $S$  and whose simplices are the non-empty subsets  $T \subset S$  for which the special subgroup  $W_T$  is finite.

Given a Coxeter system  $(W, S)$  the Davis Complex  $\Sigma = \Sigma(W, S)$  is a contractible simplicial complex on which  $W$  acts with finite stabilisers; the action of the fundamental generators  $S$  is by reflections. This complex has been introduced in [3] and it can be interpreted as the barycentric subdivision of a cell complex where the cells are in bijective correspondence with the cosets of finite special subgroups of  $W$ . This cell complex admits in a natural way a piecewise Euclidean metric and this metric can be shown to be CAT(0). The links of the 0-cells of this complex can be identified with the nerve  $L$ . The subcomplex generated by the cells corresponding to the maximal finite special subgroups is denoted by  $K$ . It is a fundamental domain of the action of  $W$  and it can be realised as the cone of  $L$ , where  $L$  is identified with the boundary  $\partial K$  in  $\Sigma$ . For details see [4].

If  $(W, S)$  is a right angled Coxeter system, then its nerve is a flag complex [4, p. 125]. Conversely, if we are given a finite flag complex  $L$ , then we can construct a Coxeter system  $(W, S)$  such that  $L$  is its nerve as follows: let  $S$  be the set of vertices of  $L$  and for  $s \neq t$  set  $m_{st} = 2$  if  $s$  and  $t$  are adjacent in  $L$  and set  $m_{st} = \infty$  if no edge connects  $s$  and  $t$  in  $L$ .

## 3. THE NON-HYPERBOLIC CASE

It suffices to show that  $\underline{\text{cd}} W \geq 3$ . For this it is enough to show that  $W$  contains a subgroup  $H$  with  $\underline{\text{cd}} H \geq 3$ . Since  $W$  is not word hyperbolic it contains a subgroup isomorphic to  $\mathbb{Z}^2$  [4, p. 241].

We show that  $\underline{\text{E}}\mathbb{Z}^2 = 3$  using an explicit 3-dimensional model  $X$  for  $\underline{\text{E}}\mathbb{Z}^2$ , which was first described by Farrell. See [8] for a general construction containing this as a special case, or see [11] for a description of  $X$  and a computation of  $H_*(X/\mathbb{Z}^2; \mathbb{Z})$  from which it follows that  $H^3(X/\mathbb{Z}^2; \mathbb{Z})$  is a countable direct product of copies of  $\mathbb{Z}$ . Theorem 4.2 in [9, p. 83] states that  $H^3(X/\mathbb{Z}^2) \cong H_{\text{vc}}^3(\mathbb{Z}^2; \mathbb{Z})$ . Hence it follows that  $\underline{\text{cd}} \mathbb{Z}^2 = 3$ .

## 4. THE GEOMETRIC DIMENSION IN THE HYPERBOLIC CASE

Given a Coxeter system  $(W, S)$  and a  $W$ -space  $X$  we set

$$X^\# = \bigcup_{s \in S} X^s$$

and

$$X^{\text{sing}} = \{x \in X \mid W_x \neq 1\}.$$

Clearly  $X^\# \subset X^{\text{sing}}$ .

**Lemma 1.** *Let  $K \subset \Sigma$  be the fundamental chamber of  $\Sigma$  and let  $s \in S$ . Then both  $K$  and  $K \cup sK$  are convex subsets of  $\Sigma$ .*

*Proof.* For each  $t \in S$  the fixedpoint set  $\Sigma^t$  separates  $\Sigma$  into two connected half spaces. Denote by  $H_t^-$  the half space which does not intersect  $K$  and denote by  $\overline{H}_t^+$  the complement of  $H_t^-$ . Then  $\overline{H}_t^+$  is a convex subset of  $\Sigma$  containing  $K$ . Then  $K = \bigcap_{t \in S} \overline{H}_t^+$  is a convex subset of  $\Sigma$ . Finally,  $K \cap sK$  is convex since  $K \cup sK = K_0 \cap sK_0$  where  $K_0$  is the convex set  $K_0 = \bigcap_{t \in S \setminus \{s\}} \overline{H}_t^+$ .  $\square$

**Lemma 2.** *Let  $X$  be a model for  $\underline{EW}$ . Then  $X^\#$  is homotopy equivalent to  $L$ .*

*Proof.* Since  $X$  is  $W$ -homotopy equivalent to  $\Sigma$  it follows that  $X^\#$  is homotopy equivalent to  $\Sigma^\#$ . Thus it is enough that  $\Sigma^\#$  is homotopy equivalent to  $L$ .

Let  $K$  be the fundamental chamber of  $\Sigma$ . Then  $K$  is complete and compact and due to Lemma 1 also convex. Therefore, since  $\Sigma$  is a CAT(0) space, there exists a retraction of  $\Sigma$  onto  $K$  which sends every point  $x \in \Sigma \setminus K$  to the unique point  $\pi(x)$  of  $K$  which is nearest to  $x$ , cf. [2, p.176f.].

Let  $K^S$  the union of all mirrors of  $K$ , that is

$$K^S = \{x \in K \mid x \in K \cap sK \text{ for some } s \in S\},$$

cf. [4, p. 63, p. 127]. The set  $K^S$  is homotopy equivalent to  $L$  [4, p. 127].

Let  $s \in S$  and  $x \in X^s \setminus K$ . Let  $y = \pi(x)$ . Then  $sy \in sK$  and since  $K \cup sK$  is convex it follows that the midpoint  $m$  of the geodesic joining  $y$  and  $sy$  is contained in  $K \cup sK$ . Since  $y$  and  $sy$  have the same distance from  $K \cap sK$  it follows that  $m \in K \cap sK$ . In particular  $m \in K$ . Since  $x \in X^s$  it follows that  $d(x, y) = d(sx, sy) = d(x, sy)$ . Since the metric of  $\Sigma$  is CAT(0) it follows that  $d(x, m) \leq \max(d(x, y), d(x, sy)) = d(x, y)$ . By the uniqueness of the point  $\pi(x)$  it follows that  $m = y$ . Hence  $y \in K^S$ .

It follows that the homotopy equivalence  $\Sigma \simeq K$  restricts to a homotopy equivalence  $\Sigma^\# \simeq K^S$ . Thus  $X^\# \simeq L$ .  $\square$

**Remark 3.** The above lemma could be used to give a slightly different proof of the main assertion of Proposition 4 of [1, p. 497].

**Lemma 4.** *Let  $X$  be a model for  $\underline{EW}$ . If  $W$  is word hyperbolic, then  $X^\#$  is homotopy equivalent to*

$$L \vee \bigvee_{i \in I} S^1$$

where the index set  $I$  consists of all maximal infinite virtually cyclic subgroups of  $W$  which contain at least two non-commuting Coxeter generators.

*Proof.* Let  $Y$  be the model for  $\underline{EW}$  which is obtained from  $\Sigma$  as described in [11]. This construction yields for every maximal infinite virtually cyclic subgroup  $H$  of  $W$  a 1-dimensional model  $Z_H$  for  $\underline{EH}$  together with an  $H$ -equivariant embedding  $f_H: Z_H \rightarrow \Sigma$ . We identify  $Z_H$  with its image in  $\Sigma$  under this embedding. Then  $Y$  is obtained by coning of the sets  $Z_H$  and extending the  $W$ -action suitably.

Since  $X$  is  $W$ -homotopy equivalent to  $Y$  it follows that  $X^\#$  is homotopy equivalent to  $Y^\#$ . The set  $Y^\#$  is obtained from  $\Sigma^\#$  by coning of the intersection  $\Sigma^\# \cap Z_H$  for every maximal infinite virtually cyclic subgroup  $H$  of  $W$ .

Let  $s, t \in S$  such that  $s, t \in H$  for some maximal infinite virtually cyclic subgroup  $H$  of  $W$ . Then  $x \in Z_H$  can be a common fixed point of  $s$  and  $t$  if and only if  $s$  and  $t$  commute. In particular  $Z_H \cap X^\#$  can consist of at most 2 points as a virtually cyclic subgroup of  $W$  cannot contain more than 2 pairwise non-commuting Coxeter generators. Coning of a singleton set of path connected space does not change its homotopy type. And coning of a subset of a path connected space which has two points is homotopy equivalent to attaching a  $S^1$  to it. Hence the claim of the lemma follows.  $\square$

**Lemma 5.** *Let  $(X, A)$  be CW-pair complexes and let  $B$  be a CW-complex which is homotopy equivalent to  $A$ . Then there exists a CW-pair  $(Y, B)$  which is homotopy equivalent to  $(X, A)$  such that the cells of  $X \setminus A$  are dimension wise in a 1-to-1 correspondence to the cells of  $Y \setminus B$ .*

*Proof.* This follows directly from Theorem 4.1.7 in [10, p. 104].  $\square$

**Theorem 6.** *Let  $(W, S)$  be a Coxeter system with  $W$  word hyperbolic and such that the nerve  $L(W, S)$  of this Coxeter system is an acyclic complex, which is not homotopy equivalent to a subcomplex of a contractible 2-complex. Then  $\underline{\underline{gd}}W = 3$ .*

*Proof.* Assume towards a contradiction that there exists a 2-dimensional model  $X$  for  $\underline{EW}$ . Then  $X^\#$  is homotopy equivalent to  $L \vee \vee S^1$  by Lemma 4. By Lemma 5 there exists a 2-dimensional CW-complex  $Y$  which is homotopy equivalent to  $X$  and which contains  $L \vee \vee S^1$ . In particular  $L$  is a subcomplex of  $Y$  contradicting the assumption that  $L$  does not embed into a contractible 2-complex. Thus  $\underline{\underline{gd}}W \geq 3$ .

On the other hand, the Davis complex  $\Sigma$  is a model for  $\underline{EW}$  and  $\dim \Sigma = \dim L + 1 = 3$ . Since  $W$  is word hyperbolic we can elevate  $\Sigma$  to a model for  $\underline{EW}$  by attaching orbits of cells in dimension 2 and less, cf. [11]. Thus  $\underline{\underline{gd}}W \leq 3$  and equality holds.  $\square$

## 5. THE COHOMOLOGICAL DIMENSION

**Theorem 7.** *Let  $(W, S)$  be a Coxeter system with  $W$  word hyperbolic and such that the nerve  $L(W, S)$  of this Coxeter system is an acyclic complex which is not homotopy equivalent to a subcomplex of a contractible 2-complex. Then  $\underline{\underline{cd}}W = 2$ .*

*Proof.* Let  $\mathfrak{F}$  be the family of virtually cyclic subgroups of  $W$ . Let  $Z$  be the submodule of the trivial  $\mathcal{O}_{\mathfrak{F}}W$ -module given by  $Z(G/H) = \mathbb{Z}$  for any finite subgroup  $H$  of  $W$  and which is 0 otherwise. The complex  $\Sigma^{\text{sing}}$  is acyclic and 2-dimensional by [1] and it follows that  $\underline{C}_*(\Sigma^{\text{sing}})$  gives a projective resolution of  $Z$  of length 2. Thus  $\text{pd}Z \leq 2$ .

On the other hand, if  $X$  is a model for  $\underline{EW}$ , then a model  $Y$  for  $\underline{EW}$  can be obtained from  $X$  by attaching orbits of cells in dimension 2 and less [11, Proposition 9]. It follows that  $\underline{C}_*(Y, X)$  gives a free resolution of  $Q = \mathbb{Z}/Z$  of length 2. Thus  $\text{pd}Q \leq 2$ .

Consider the short exact sequence

$$0 \rightarrow Z \rightarrow \mathbb{Z} \rightarrow Q \rightarrow 0$$

of  $\mathcal{O}_{\mathbb{F}}W$ -modules. Since  $\text{pd}Z$  and  $\text{pd}Q$  are bounded by 2 it follows by the Horseshoe Lemma that  $\underline{\text{pd}}\mathbb{Z} \leq 2$ , that is  $\underline{\text{cd}}W \leq 2$ .

On the other hand, it follows from [11, Corollary 16] that the quotient space  $\underline{E}W/W$  has non-trivial cohomology in dimension 2, and thus  $H_{\mathbb{F}}^2(W; \mathbb{Z})$  must be non-trivial too, cf. Theorem 4.2 in [9, p. 83]. As a consequence we get  $\underline{\text{cd}}W \geq 2$  and therefore the claim follows.  $\square$

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