

A note on equilibrium Glauber and Kawasaki dynamics for permanental point processes

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Abstract

We construct two types of equilibrium dynamics of an infinite particle system in a locally compact metric space X for which a permanental point process is a symmetrizing, and hence invariant measure. The Glauber dynamics is a birth-and-death process in X , while in the Kawasaki dynamics interacting particles randomly hop over X . In the case $X = \mathbb{R}^d$, we consider a diffusion approximation for the Kawasaki dynamics at the level of Dirichlet forms. This leads us to an equilibrium dynamics of interacting Brownian particles for which a permanental point process is a symmetrizing measure.

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1 Introduction

Let X be a locally compact Polish space and let ν be a Radon non-atomic measure on it. Let $\Gamma = \Gamma_X$ denote the space of all locally finite subsets (configurations) in X .

A Glauber dynamics (a birth-and-death process of an infinite system of particles in X) is a Markov process on Γ whose formal (pre-)generator has the form

$$\begin{aligned} (L_G F)(\gamma) = & \sum_{x \in \gamma} d(x, \gamma \setminus x) (F(\gamma \setminus x) - F(\gamma)) \\ & + \int_X \nu(dx) b(x, \gamma) (F(\gamma \cup x) - F(\gamma)), \quad \gamma \in \Gamma. \end{aligned} \tag{1.1}$$

Here and below, for simplicity of notation we write x instead of $\{x\}$. The coefficient $d(x, \gamma \setminus x)$ describes the rate at which particle x of configuration γ dies, while $b(x, \gamma)$ describes the rate at which, given configuration γ , a new particle is born at x .

A Kawasaki dynamics (a dynamics of hopping particles) is a Markov process on Γ whose formal (pre-)generator is

$$(L_K F)(\gamma) = \sum_{x \in \gamma} c(x, y, \gamma \setminus x) (F(\gamma \setminus x \cup y) - F(\gamma)), \quad \gamma \in \Gamma. \quad (1.2)$$

The coefficient $c(x, y, \gamma \setminus x)$ describes the rate at which particle x of configuration γ hops to y , taking the rest of the configuration, $\gamma \setminus x$, into account.

Equilibrium Glauber and Kawasaki dynamics which have a standard Gibbs measure as symmetrizing (and hence invariant) measure were constructed in [19, 20]. In [22], this construction was extended to the case of an equilibrium dynamics which has a determinantal (fermion) point process as invariant measure. For further studies of equilibrium and non-equilibrium Glauber and Kawasaki dynamics, we refer to [3, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 21, 28] and the references therein.

The aim of this note is to show that general criteria of existence of Glauber and Kawasaki dynamics which were developed in [22] are applicable to a wide class of α -permanental ($\alpha \in \mathbb{N}$) point processes, proposed by Shirai and Takahashi [30]. This class includes classical permanental (boson) point processes, see e.g. [5, 30]. We will also consider a diffusion approximation for the Kawasaki dynamics at the level of Dirichlet forms (compare with [15]). This will lead us to an equilibrium dynamics of interacting Brownian particles for which an α -permanental point process is a symmetrizing measure. As a by-product of our considerations, we will also extend the result of [30] on the existence of α -permanental point process.

2 Equilibrium Glauber and Kawasaki dynamics – general results

Let X be a locally compact Polish space. We denote by $\mathcal{B}(X)$ the Borel σ -algebra on X , and by $\mathcal{B}_0(X)$ the collection of all sets from $\mathcal{B}(X)$ which are relatively compact. We fix a Radon, non-atomic measure on $(X, \mathcal{B}(X))$. (For most applications, the reader may think of X as \mathbb{R}^d and ν as the Lebesgue measure.)

The configuration space Γ over X is defined as the set of all subsets of X which are locally finite

$$\Gamma := \{ \gamma \subset X : |\gamma_\Lambda| < \infty \text{ for each } \Lambda \in \mathcal{B}_0(X) \},$$

where $|\cdot|$ denotes the cardinality of a set and $\gamma_\Lambda := \gamma \cap \Lambda$. One can identify any $\gamma \in \Gamma$ with the positive Radon measure $\sum_{x \in \gamma} \varepsilon_x$, where ε_x is the Dirac measure with mass at x and $\sum_{x \in \emptyset} \varepsilon_x :=$ zero measure. The space Γ can be endowed with the vague topology, i.e., the weakest topology on Γ with respect to which all maps

$$\Gamma \ni \gamma \mapsto \langle \varphi, \gamma \rangle := \int_X \varphi(x) \gamma(dx) = \sum_{x \in \gamma} \varphi(x), \quad \varphi \in C_0(X),$$

are continuous. Here, $C_0(X)$ is the space of all continuous, real-valued functions on X with compact support. We denote the Borel σ -algebra on Γ by $\mathcal{B}(\Gamma)$. A point process in X is a probability measure on $(\Gamma, \mathcal{B}(\Gamma))$.

We fix a point process μ which satisfies the so-called condition (Σ'_ν) [5, 26], i.e., there exist a measurable function $r : X \times \Gamma \rightarrow [0, +\infty]$, called the Papangelou intensity of μ , such that

$$\int_{\Gamma} \mu(d\gamma) \int_X \gamma(dx) F(x, \gamma) = \int_{\Gamma} \mu(d\gamma) \int_X \nu(dx) r(x, \gamma) F(x, \gamma \cup x) \quad (2.1)$$

for any measurable function $F : X \times \Gamma \rightarrow [0, +\infty]$. The condition (Σ'_ν) can be thought of as a kind of weak Gibbsianity of μ . Intuitively, we may treat the Papangelou intensity as

$$r(x, \gamma) = \exp[-E(x, \gamma)], \quad (2.2)$$

where $E(x, \gamma)$ is the relative energy of interaction between particle x and configuration γ .

To define an equilibrium Glauber dynamics for which μ is a symmetrizing measure, we fix a death coefficient as a measurable function $d : X \times \Gamma \rightarrow [0, +\infty]$, and then define a birth coefficient $b : X \times \Gamma \rightarrow [0, +\infty]$ by

$$b(x, \gamma) = d(x, \gamma)r(x, \gamma), \quad (x, \gamma) \in X \times \Gamma. \quad (2.3)$$

To define a Kawasaki dynamics, we fix a measurable function $c : X^2 \times \Gamma^2 \rightarrow [0, +\infty]$ which satisfies

$$r(x, \gamma)c(x, y, \gamma) = r(y, \gamma)c(y, x, \gamma), \quad (x, y, \gamma) \in X^2 \times \Gamma. \quad (2.4)$$

Formulas (2.3) and (2.4) are called the balance conditions [13, 14]. We will also assume that the function $c(x, y, \gamma)$ vanishes if at least one of the functions $r(x, \gamma)$ and $r(y, \gamma)$ vanishes, i.e.,

$$c(x, y, \gamma) = c(x, y, \gamma)\chi_{\{r>0\}}(x, \gamma)\chi_{\{r>0\}}(y, \gamma). \quad (2.5)$$

Here, for a set A , χ_A denotes the indicator function of A . We refer to [22, Remark 3.1] for a justification of this assumption, which involves the interpretation of $r(x, \gamma)$ as in (2.2), see also Remark 2.4 below.

We denote by $\mathcal{FC}_b(C_0(X), \Gamma)$ the space of all functions of the form

$$\Gamma \ni \gamma \mapsto F(\gamma) = g(\langle \varphi_1, \gamma \rangle, \dots, \langle \varphi_N, \gamma \rangle), \quad (2.6)$$

where $N \in \mathbb{N}$, $\varphi_1, \dots, \varphi_N \in C_0(X)$ and $g \in C_b(\mathbb{R}^N)$. Here, $C_b(\mathbb{R}^N)$ denotes the space of all continuous bounded functions on \mathbb{R}^N . We assume that, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \gamma(dx) d(x, \gamma \setminus x) < \infty, \quad (2.7)$$

$$\int_{\Gamma} \mu(d\gamma) \int_X \gamma(dx) \int_X \nu(dy) c(x, y, \gamma \setminus x) (\chi_{\Lambda}(x) + \chi_{\Lambda}(y)) < \infty. \quad (2.8)$$

As easily seen, conditions (2.7) and (2.8) are sufficient in order to define bilinear forms

$$\begin{aligned} \mathcal{E}_G(F, G) &:= \int_{\Gamma} \mu(d\gamma) \int_X \gamma(dx) d(x, \gamma \setminus x) (F(\gamma \setminus x) - F(\gamma))(G(\gamma \setminus x) - G(\gamma)), \\ \mathcal{E}_K(F, G) &:= \frac{1}{2} \int_{\Gamma} \mu(d\gamma) \int_X \gamma(dx) \int_X \nu(dy) c(x, y, \gamma \setminus x) (F(\gamma \setminus x \cup y) - F(\gamma)) \\ &\quad \times (G(\gamma \setminus x \cup y) - G(\gamma)), \end{aligned}$$

where $F, G \in \mathcal{F}C_b(C_0(X), \Gamma)$.

For the construction of the Kawasaki dynamics, we will also assume that the following technical assumptions holds:

$$\begin{aligned} \exists u, v \in \mathbb{R} \quad \forall \Lambda \in \mathcal{B}_0(X) : \\ \int_{\Lambda} \gamma(dx) \int_{\Lambda} \nu(dy) r(x, \gamma \setminus x)^u r(y, \gamma \setminus x)^v c(x, y, \gamma \setminus y) \in L^2(\Gamma, \mu) < \infty. \end{aligned} \quad (2.9)$$

Note that in formula (2.9) and below, we use the convention $\frac{0}{0} := 0$.

The following theorem was essentially proved in [22].

Theorem 2.1. *(i) Assume that a point process μ satisfies (2.1). Assume that conditions (2.3), (2.7), respectively (2.4), (2.5), (2.8), and (2.9) are satisfied. Let $\sharp = G, K$. Then the bilinear form $(\mathcal{E}_{\sharp}, \mathcal{F}C_b(C_0(x), \Gamma))$ is closable in $L^2(\Gamma, \mu)$ and its closure will be denoted by $(\mathcal{E}_{\sharp}, D(\mathcal{E}_{\sharp}))$. Further there exists a conservative Hunt process (Glauber, respectively Kawasaki dynamics)*

$$M^{\sharp} = \left(\Omega^{\sharp}, \mathcal{F}^{\sharp}, (\mathcal{F}_t^{\sharp})_{t \geq 0}, (\Theta_t^{\sharp})_{t \geq 0}, (X^{\sharp}(t))_{t \geq 0}, (P_{\gamma}^{\sharp})_{\gamma \in \Gamma} \right)$$

on Γ which is properly associated with $(\mathcal{E}_{\sharp}, D(\mathcal{E}_{\sharp}))$, i.e., for all (μ -version of) $F \in L^2(\Gamma, \mu)$ and $t > 0$

$$\Gamma \ni \gamma \mapsto p_t^{\sharp} F(\gamma) := \int_{\Omega^{\sharp}} F(X^{\sharp}(t)) dP_{\gamma}^{\sharp}$$

is an \mathcal{E}^{\sharp} -quasi continuous version of $\exp(tL_{\sharp})F$, where $(-L_{\sharp}, D(L_{\sharp}))$ is the generator of $(\mathcal{E}_{\sharp}, D(\mathcal{E}_{\sharp}))$. M^{\sharp} is up-to μ -equivalence unique. In particular, M^{\sharp} is μ -symmetric and has μ as invariant measure.

(ii) M^{\sharp} from (i) is up to μ -equivalence unique between all Hunt processes

$$M' = \left(\Omega', \mathcal{F}', (\mathcal{F}_t')_{t \geq 0}, (\Theta_t')_{t \geq 0}, (X'(t))_{t \geq 0}, (P'_{\gamma})_{\gamma \in \Gamma} \right)$$

on Γ having μ as invariant measure and solving a martingale problem for $(L_{\sharp}, D(L_{\sharp}))$, i.e., for all $G \in D(H_{\sharp})$

$$\tilde{G}(X'(t)) - \tilde{G}(X'(0)) - \int_0^t (L_{\sharp}G)(X'(s)) ds, \quad t \geq 0,$$

is an (\mathcal{F}_t^l) -martingale under P'_γ for $\mathcal{E}_\#$ -q.e. $\gamma \in \Gamma$. Here, \tilde{G} denotes an $\mathcal{E}_\#$ -quasi-continuous version of G .

(iii) Further assume that, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\int_\Lambda \gamma(dx) d(x, \gamma \setminus x) \in L^2(\Gamma, \mu), \quad \int_\Lambda \nu(dx) b(x, \gamma) \in L^2(\Gamma, \mu), \quad (2.10)$$

in the Glauber case, and

$$\int_X \gamma(dx) \int_X \nu(dy) c(x, y, \gamma \setminus x)(\chi_\Lambda(x) + \chi_\Lambda(y)) \in L^2(\Gamma, \mu) \quad (2.11)$$

in the Kawasaki case. Then $\mathcal{FC}_b(C_0(X), \Gamma) \subset D(L_\#)$, and for each $F \in \mathcal{FC}_b(C_0(X), \Gamma)$, $L_\#F$ is given by formulas (1.1) and (1.2), respectively.

Remark 2.1. We refer to [24] for an explanation of notions appearing in Theorem 2.1, see also a brief explanation of them in [22].

Proof of Theorem 2.1. The statement follows from Theorems 3.1 and 3.2 in [22]. Note that, although these theorems are formulated for determinantal point processes only, their proof only uses the (Σ'_ν) property of these point processes. Note also that condition (2.9) is formulated in [22] only for $v = 1$, however the proof of Lemma 3.2 in [22] admits a straightforward generalization to the case of an arbitrary $v \in \mathbb{R}$. \square

Remark 2.2. Part (iii) of Theorem 2.1 states that the operator $(-L_\#, D(L_\#))$ is the Friedrichs' extension of the operator $(-L_\#, \mathcal{FC}_b(C_0(X), \Gamma))$ defined by formulas (1.1), (1.2), respectively.

Let us fix a parameter $s \in [0, 1]$ and define

$$d(x, \gamma) := r(x, \gamma)^{s-1} \chi_{\{r>0\}}(x, \gamma), \quad (x, \gamma) \in X \times \Gamma, \quad (2.12)$$

$$b(x, \gamma) := r(x, \gamma)^s \chi_{\{r>0\}}(x, \gamma), \quad (x, \gamma) \in X \times \Gamma, \quad (2.13)$$

$$c(x, y, \gamma) := a(x, y) r(x, \gamma)^{s-1} r(y, \gamma)^s \chi_{\{r>0\}}(x, \gamma) \chi_{\{r>0\}}(y, \gamma), \quad (x, y, \gamma) \in X^2 \times \Gamma. \quad (2.14)$$

Here the function $a : X^2 \rightarrow [0, +\infty)$ is bounded, measurable, symmetric (i.e., $a(x, y) = a(y, x)$), and satisfies

$$\sup_{x \in X} \int_X a(x, y) \nu(dy) < \infty. \quad (2.15)$$

Note that the balance conditions (2.3) and (2.4) are satisfied for these coefficients, and so is condition (2.5).

Remark 2.3. Note that, if $X = \mathbb{R}^d$ and $a(x, y)$ has the form $a(x - y)$ for a function $a : \mathbb{R}^d \rightarrow [0, \infty)$, then condition (2.15) means that $a \in L^1(\mathbb{R}^d, dx)$. (Here and below, in the case $X = \mathbb{R}^d$, we use an obvious abuse of notation.)

Remark 2.4. Using representation (2.2), we can rewrite formulas (2.12)–(2.14) as follows:

$$\begin{aligned} d(x, \gamma \setminus x) &= \exp[(1-s)E(x, \gamma \setminus x)]\chi_{\{E < +\infty\}}(x, \gamma \setminus x), \\ b(x, \gamma \setminus x) &= \exp[-sE(x, \gamma \setminus x)]\chi_{\{E < +\infty\}}(x, \gamma \setminus x), \\ c(x, y, \gamma \setminus x) &= a(x, y) \exp[(1-s)E(x, \gamma \setminus x) - sE(y, \gamma \setminus x)] \\ &\quad \times \chi_{\{E < +\infty\}}(x, \gamma \setminus x)\chi_{\{E < +\infty\}}(y, \gamma \setminus x). \end{aligned}$$

So, if the corresponding dynamics exist, one can give the following heuristic description of them: Both dynamics are concentrated on configurations $\gamma \in \Gamma$ such that, for each $x \in \gamma$, the relative energy of interaction between x and the rest of configuration, $\gamma \setminus x$, is finite; those particles tend to die, respectively hop, which have a high energy of interaction with the rest of the configuration, while it is more probable that a new particle is born at y , respectively x hops to y , if the energy of interaction between y and the rest of the configuration is low.

Let us assume that the point process μ satisfies:

$$\forall \Lambda \in \mathcal{B}_0(X) : \int_{\Lambda} \gamma(dx) \in L^2(\Gamma, \mu).$$

Then, by choosing $u = 1 - s$ and $v = -s$ in (2.9), we conclude that the coefficient c given by (2.14) satisfies (2.9).

We will construct below a class of point processes μ for which the coefficients d , b and c given above satisfy the other conditions of Theorem 2.1.

3 Permanent point processes and corresponding equilibrium dynamics

Let K be a linear, bounded, self-adjoint operator on the real space $L^2(X, \nu)$. Further assume that $K \geq 0$ and K is locally of trace class, i.e., $\text{Tr}(P_{\Lambda} K P_{\Lambda}) < \infty$ for all $\Lambda \in \mathcal{B}_0(X)$, where P_{Λ} denotes the operator of multiplication by χ_{Λ} . Hence, each operator $P_{\Lambda} \sqrt{K}$ is of Hilbert–Schmidt class. Following [23] (see also [12, Lemma A.4]), we conclude that \sqrt{K} is an integral operator whose integral kernel, $\varkappa(x, y)$, satisfies

$$\int_{\Lambda} \int_X \nu(dx) \nu(dy) \varkappa(x, y)^2 < \infty \quad \text{for all } \Lambda \in \mathcal{B}_0(X). \quad (3.1)$$

In particular,

$$\varkappa(x, \cdot) \in L^2(X, \nu) \quad \text{for } \nu\text{-a.a. } x \in X. \quad (3.2)$$

Hence, K is an integral operator whose integral kernel can be chosen as

$$\begin{aligned} k(x, y) &= \int_X \varkappa(x, z)\varkappa(z, y)\nu(dz) \\ &= \int_X \varkappa(x, z)\varkappa(y, z)\nu(dz) = (\varkappa(x, \cdot), \varkappa(y, \cdot))_{L^2(X, \nu)}. \end{aligned} \quad (3.3)$$

We also have, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\begin{aligned} \text{Tr}(P_\Lambda K P_\Lambda) &= \|\sqrt{K}P_\Lambda\|_{\text{HS}}^2 \\ &= \int_\Lambda \nu(dx) \int_X \nu(dy) \varkappa(x, y)^2 = \int_\Lambda k(x, x) \nu(dx), \end{aligned} \quad (3.4)$$

where $\|\cdot\|_{\text{HS}}$ denotes the Hilbert–Schmidt norm.

Proposition 3.1. *There exists a random field $(Y(x))_{x \in X}$ on a probability space (Ω, \mathcal{A}, P) such that the mapping*

$$X \times \Omega \ni (x, \omega) \mapsto Y(x, \omega) \quad (3.5)$$

is measurable, and for ν -a.a. $x \in X$, $Y(x)$ is a Gaussian random variable with mean 0 and such that

$$\mathbb{E}(Y(x)Y(y)) = k(x, y) \quad \text{for } \nu^{\otimes 2}\text{-a.a. } (x, y) \in X^2 \text{ and } \nu\text{-a.a. } x = y \in X. \quad (3.6)$$

Remark 3.1. The statement of Proposition 3.1 is well-known if the integral kernel of the operator K admits a continuous version (see e.g. Theorem 1.8 and p. 456 in [30]). In the latter case, $(Y(x))_{x \in X}$ is a Gaussian random field and formula (3.6) holds for all $(x, y) \in X^2$.

Proof of Proposition 3.1. Consider a standard triple of real Hilbert spaces

$$H_+ \subset H_0 = L^2(X, \nu) \subset H_-.$$

Here the Hilbert space H_+ is densely and continuously embedded into H_0 , the inclusion operator $H_+ \hookrightarrow H_0$ is of Hilbert–Schmidt class, and the Hilbert space H_- is the dual space of H_+ with respect to the center space H_0 (see e.g. [2]).

Let \mathbb{P} be the standard Gaussian measure on H_- , i.e., the probability measure on the Borel σ -algebra $\mathcal{B}(H_-)$ which has Fourier transform

$$\int_{H_-} e^{i\langle \omega, f \rangle} \mathbb{P}(d\omega) = \exp\left[-\frac{1}{2}\|f\|_{H_0}^2\right], \quad f \in H_+,$$

where $\langle \omega, f \rangle$ denotes the dual pairing between $\omega \in H_-$ and $f \in H_+$. Then the mapping $H_+ \ni f \rightarrow \langle \cdot, f \rangle$ can be extended by continuity to an isometry

$$I : H_0 \rightarrow L^2(H_-, \mathbb{P}). \quad (3.7)$$

For any $f \in H_0$ we denote $\langle \cdot, f \rangle := If$. Thus, for each $f \in H_0$, $\langle \cdot, f \rangle$ is a (complex) Gaussian random variable with mean 0 and for any $f, g \in H_0$

$$\int_{H_-} \langle \omega, f \rangle \langle \omega, g \rangle \mathbb{P}(d\omega) = (f, g)_{L^2(X, \nu)}. \quad (3.8)$$

Thus, by (3.2), we set for ν -a.a. $x \in X$, $\tilde{Y}(x, \omega) := \langle \omega, k(x, \cdot) \rangle$. Hence $\tilde{Y}(x)$ is a Gaussian random variable and by (3.3) and (3.8), (3.6) holds.

Hence, it remains to prove that there exists a random field $Y = (Y(x))_{x \in X}$ for which the mapping (3.5) is measurable and such that $Y(x, \omega) = \tilde{Y}(x, \omega)$ for $\nu \otimes \mathbb{P}$ -a.a. (x, ω) . To this end, we fix any $\Lambda \in \mathcal{B}_0(X)$ and denote by $\mathcal{B}(\Lambda)$ the trace σ -algebra of $\mathcal{B}(X)$ on Λ . We define a set \mathcal{D}_Λ of the functions $u : \Lambda \times X \rightarrow \mathbb{R}$ of the form

$$u(x, y) = \sum_{i=1}^n \chi_{\Delta_i}(x) f_i(y), \quad (3.9)$$

where $\Delta_i \in \mathcal{B}(\Lambda)$, $f_i \in H_+$, $i = 1, \dots, n$. Define a linear mapping

$$I_\Lambda : \mathcal{D}_\Lambda \rightarrow L^2(\Lambda \times H_-, \nu \otimes \mathbb{P}) \quad (3.10)$$

by setting, for each $u \in \mathcal{D}_\Lambda$ of the form (3.9),

$$(I_\Lambda u)(x, \omega) = \sum_{i=1}^n \chi_{\Delta_i}(x) \langle \omega, f_i \rangle, \quad (x, \omega) \in \Lambda \times H_-.$$

Clearly, I_Λ can be extended to an isometry

$$I_\Lambda : L^2(\Lambda \times X, \nu^{\otimes 2}) \rightarrow L^2(\Lambda \times H_-, \nu \otimes \mathbb{P}),$$

and we have $I_\Lambda = \mathbf{1}_\Lambda \otimes I$, where $\mathbf{1}_\Lambda$ is the identity operator in $L^2(\Lambda, \nu)$ and the operator I is as in (3.7).

Fix any $u \in L^2(\Lambda \times X, \nu^{\otimes 2})$. As easily seen, there exist a sequence $(u_n)_{n=1}^\infty \subset \mathcal{D}_\Lambda$ such that $u_n \rightarrow u$ in $L^2(\Lambda \times X, \nu^{\otimes 2})$ and for ν -a.a. $x \in \Lambda$, $u_n(x, \cdot) \rightarrow u(x, \cdot)$ in $L^2(X, \nu)$. Hence, for ν -a.a. $x \in \Lambda$, $I_\Lambda u_n(x, \cdot) \rightarrow I_\Lambda u(x, \cdot)$ in $L^2(H_-, \mathbb{P})$, which implies

$$(I_\Lambda u)(x, \omega) = \langle \omega, u(x, \cdot) \rangle \quad \text{for } \mathbb{P}\text{-a.a. } \omega \in H_-. \quad (3.11)$$

Now, denote by \varkappa_Λ the restriction of \varkappa to the set $\Lambda \times X$. For ν -a.a. $x \in \Lambda$, we define $Y_\Lambda(x) := (I_\Lambda \varkappa_\Lambda)(x, \cdot)$. Hence, by (3.11), for ν -a.a. $x \in \Lambda$, $Y_\Lambda(x) = \tilde{Y}(x)$ \mathbb{P} -a.e. Finally, let $(\Lambda_n)_{n=1}^\infty \subset \mathcal{B}_0(X)$ be such that $\Lambda_n \cap \Lambda_m = \emptyset$ if $n \neq m$ and $\bigcup_{n=1}^\infty \Lambda_n = X$. Setting $Y(x) := Y_{\Lambda_n}(x)$ for ν -a.a. $x \in \Lambda_n$, $n \in \mathbb{N}$, we conclude the statement. \square

Let Y be a random field as in Proposition 3.1. For each $\Lambda \in \mathcal{B}_0(X)$, we have

$$\begin{aligned} \mathbb{E} \left(\int_{\Lambda} Y(x)^2 \nu(dx) \right) &= \int_{\Lambda} \mathbb{E}(Y(x)^2) \nu(dx) \\ &= \int_{\Lambda} \nu(dx) \int_X \nu(dy) \kappa(x, y)^2 < \infty. \end{aligned}$$

In particular, the function $Y(x)^2$ is locally ν -integrable \mathbb{P} -a.s. Let $l \in \mathbb{N}$ and let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space on which l independent copies Y_1, Y_2, \dots, Y_l of a random field Y as in Proposition 3.1 are defined. Denote by $\mu^{(l)}$ the Cox point process on X with random intensity $g^{(l)}(x) = \sum_{i=1}^l Y_i(x)^2$, which is locally ν -integrable \mathbb{P} -a.s. Thus, $\mu^{(l)}$ is the probability measure on $(\Gamma, \mathcal{B}(\Gamma))$ which satisfies

$$\int_{\Gamma} \mu^{(l)}(d\gamma) F(\gamma) = \int_{\Omega} \mathbb{P}(d\omega) \int_{\Gamma} \pi_{g^{(l)}(x, \omega)\nu(dx)}(d\gamma) F(\gamma) \quad (3.12)$$

for each measurable function $F : \Gamma \rightarrow [0, +\infty]$. Here, for a locally ν -integrable function $g : X \rightarrow [0, +\infty)$, we denote by $\pi_{g(x)\nu(dx)}$ the Poisson point process in X with intensity measure $g(x)\nu(dx)$, see e.g [5]. This is the unique point process in X which satisfies the Mecke identity

$$\int_{\Gamma} \pi_{g(x)\nu(dx)}(d\gamma) \int_X \gamma(dx) F(x, \gamma) = \int_{\Gamma} \pi_{g(x)\nu(dx)}(d\gamma) \int_X \nu(dx) g(x) F(x, \gamma \cup x) \quad (3.13)$$

for each measurable $F : X \times \Gamma \rightarrow [0, +\infty]$. By (3.12) and (3.13) (compare with e.g. [27]), for each $l \in \mathbb{N}$, the point process $\mu^{(l)}$ satisfies condition (Σ'_ν) and its Papangelou intensity is given by

$$r^{(l)}(x, \gamma) = \tilde{\mathbb{E}}(g^{(l)}(x) \mid \mathcal{F})(\gamma) = \tilde{\mathbb{E}}\left(\sum_{i=1}^l Y_i(x)^2 \mid \mathcal{F}\right)(\gamma). \quad (3.14)$$

Here $\tilde{\mathbb{E}}$ denotes the (conditional) expectation with respect to the probability measure

$$\tilde{\mathbb{P}}(d\omega, d\gamma) = \tilde{\mathbb{P}}(d\omega) \pi_{g^{(l)}(x, \omega)\nu(dx)}(d\gamma) \quad (3.15)$$

on $\Omega \times \Gamma$, while \mathcal{F} denotes the σ -algebra on $\Omega \times \Gamma$ generated by the mappings

$$\Omega \times \Gamma \ni (\omega, \gamma) \rightarrow F(\gamma) \in \mathbb{R},$$

where $F : \Gamma \rightarrow \mathbb{R}$ is measurable.

Recall that a point process μ in X is said to have correlation functions if, for each $n \in \mathbb{N}$, there exist a non-negative, measurable, symmetric function $k_\mu^{(n)}$ on X^n such

that, for any measurable, symmetric function $f^n : X^n \rightarrow [0, +\infty]$,

$$\begin{aligned} & \int_{\Gamma} \sum_{\{x_1, \dots, x_n\} \subset \gamma} f^n(x_1, \dots, x_n) \mu(d\gamma) \\ &= \frac{1}{n!} \int_{X^n} f^n(x_1, \dots, x_n) k_{\mu}^{(n)}(x_1, \dots, x_n) \nu(dx_1) \cdots \nu(dx_n). \end{aligned} \quad (3.16)$$

As well known (e.g. [5]), for a locally ν -integrable function $g : X \rightarrow [0, +\infty)$, the Poisson point process $\pi_{g(x)\nu(dx)}$ has correlation functions

$$k_{\mu}^{(n)}(x_1, \dots, x_n) = g(x_1) \cdots g(x_n). \quad (3.17)$$

Let us recall the notion of α -permanent [31], called α -determinant in [30]. For a square matrix $A = (a_{ij})_{i,j=1}^n$ and $\alpha \in \mathbb{R}$, we set

$$\text{per}_{\alpha} A := \sum_{\sigma \in S_n} \alpha^{n-m(\sigma)} \prod_{i=1}^n a_{i\sigma(i)},$$

where S_n is the group of all permutations of $\{1, \dots, n\}$ and $m(\sigma)$ denotes the number of cycles in σ . In particular, $\text{per}_1 A$ is the usual permanent of A , while $\text{per}_{-1} A$ is the usual determinant of A . Analogously to [30, subsec. 6.4], we conclude from (3.12), (3.16) and (3.17) that the point process $\mu^{(l)}$ has correlation functions

$$k_{\mu^{(l)}}^{(n)}(x_1, \dots, x_n) = \text{per}_{\frac{l}{2}} (lk(x_i, x_j))_{i,j=1}^n \quad \text{for } \nu^{\otimes n}\text{-a.a. } (x_1, \dots, x_n) \in X^n. \quad (3.18)$$

For $l = 2$, the point process $\mu^{(2)}$ is often called a boson point process, see e.g. [5, 23]. Thus, we have proved the following

Proposition 3.2. *For each $l \in \mathbb{N}$, there exists a point process $\mu^{(l)}$ in X whose correlation functions are given by (3.18). The $\mu^{(l)}$ satisfies condition (Σ'_ν) and its Papangelou intensity is given by (3.14).*

Remark 3.2. Recall that in [30], under the same assumptions on the operator K , the existence of a point process with correlation functions (3.18) was proved for even $l \in \mathbb{N}$, and for odd $l \in \mathbb{N}$ the statement of Proposition 3.2 was proved under the additional assumption of continuity of the integral kernel $k(\cdot, \cdot)$.

We will now prove that, for a point process $\mu^{(l)}$ as in Proposition 3.2, Glauber and Kawasaki dynamics with coefficients (2.12), (2.13) and (2.14), respectively exist.

Theorem 3.1. *(i) For each point process $\mu^{(l)}$ as in Proposition 3.2, the coefficients $d(x, \gamma)$ and $b(x, \gamma)$ defined by (2.12) and (2.13), satisfy conditions (2.3) and (2.7) and so statements (i) and (ii) of Theorem 2.1 hold, in particular, a corresponding Glauber dynamics exists.*

(ii) Assume additionally that $k(x, x)$ is bounded outside a set $\Delta \in \mathcal{B}_0(X)$. Then for a point process $\mu^{(l)}$ as in Proposition 3.2, the coefficient $c(x, y, \gamma)$ defined by (2.14), satisfies (2.4), (2.5), (2.8) and (2.9), and so statements (i) and (ii) of Theorem 2.1 hold, in particular, a corresponding Kawasaki dynamics exists.

Proof. We start with the following

Lemma 3.1. For each $n \in \mathbb{N}$ and for ν -a.a. $x \in X$

$$\int_{\Gamma} r(x, \gamma)^n \mu(d\gamma) \leq \frac{(2n)!}{2^n n!} k(x, x)^n. \quad (3.19)$$

Proof. Using Jensen's inequality for conditional expectation and the formula for moments of a Gaussian measure (see e.g. [2, Chapter 2, Section 2, Lemma 2.1]), we have

$$\begin{aligned} \int_{\Gamma} r(x, \gamma)^n \mu(d\gamma) &= \tilde{\mathbb{E}}(\tilde{\mathbb{E}}(Y(x)^2 \mid \mathcal{F})^n) \leq \tilde{\mathbb{E}}(\tilde{\mathbb{E}}(Y(x)^{2n} \mid \mathcal{F})) \\ &= \tilde{\mathbb{E}}(Y(x)^{2n}) \leq \frac{(2n)!}{2^n n!} \|\varkappa(x, \cdot)\|_{L^2(X, \nu)}^{2n} = \frac{(2n)!}{2^n n!} k(x, x)^n \end{aligned}$$

for ν -a.a. $x \in X$. □

We will only prove statement (ii) of Theorem 3.1, as the proof of statement (i) is similar and simpler. Also, for simplicity of notation, we will only consider the case $l = 1$ (for $l > 1$ the proof being similar). We will also omit the upper index (1) from our notation. By (2.1) we have, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\begin{aligned} &\int_{\Gamma} \mu(d\gamma) \int_X \gamma(dx) \int_X \nu(dy) c(x, y, \gamma \setminus x) (\chi_{\Lambda}(x) + \chi_{\Lambda}(y)) \\ &= \int_{\Gamma} \mu(d\gamma) \int_X \nu(dx) \int_X \nu(dy) r(x, \gamma) c(x, y, \gamma) (\chi_{\Lambda}(x) + \chi_{\Lambda}(y)) \\ &= \int_{\Gamma} \mu(d\gamma) \int_X \nu(dx) \int_X \nu(dy) a(x, y) r(x, \gamma)^s r(y, \gamma)^s \chi_{\{r>0\}}(x, \gamma) \\ &\quad \times \chi_{\{r>0\}}(y, \gamma) (\chi_{\Lambda}(x) + \chi_{\Lambda}(y)) \quad (3.20) \\ &\leq \int_{\Gamma} \mu(d\gamma) \int_X \nu(dx) \int_X \nu(dy) a(x, y) r(x, \gamma)^s r(y, \gamma)^s (\chi_{\Lambda}(x) + \chi_{\Lambda}(y)) \\ &= 2 \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) r(x, \gamma)^s r(y, \gamma)^s \\ &\leq 2 \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) (1 + r(x, \gamma)) (1 + r(y, \gamma)). \end{aligned}$$

By (2.15)

$$\int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) < \infty. \quad (3.21)$$

Below, $C_i, i = 1, 2, 3, \dots$, will denote positive constants whose explicit values are not important for us. We have, by (2.15)

$$\begin{aligned}
& \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) r(x, \gamma) \\
&= \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) r(x, \gamma) \left(\int_X \nu(dy) a(x, y) \right) \\
&\leq C_1 \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) r(x, \gamma) \\
&= C_1 \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \gamma(dx) = C_1 \int_{\Lambda} k(x, x) \nu(dx) < \infty.
\end{aligned} \tag{3.22}$$

Next, by (3.14)

$$\begin{aligned}
& \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) r(y, \gamma) \\
&= \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) \int_{\Gamma} \mu(d\gamma) r(y, \gamma) \\
&= \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) k(y, y) \\
&= \int_{\Lambda} \nu(dx) \int_{\Delta} \nu(dy) a(x, y) k(y, y) + \int_{\Lambda} \nu(dx) \int_{\Delta^c} \nu(dy) a(x, y) k(y, y) \\
&\leq C_2 \int_{\Lambda} \nu(dx) \int_{\Delta} \nu(dy) k(y, y) + C_3 \int_{\Lambda} \nu(dx) \int_{\Delta^c} \nu(dy) a(x, y) < \infty,
\end{aligned} \tag{3.23}$$

where we used that the function a is bounded and $k(y, y)$ is bounded on Δ^c . Analogously, using Lemma 3.1, we have

$$\begin{aligned}
& \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) r(x, \gamma) r(y, \gamma) \\
&\leq \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) \|r(x, \cdot)\|_{L^2(\mu)} \|r(y, \cdot)\|_{L^2(\mu)} \\
&\leq C_4 \int_{\Lambda} \nu(dx) \int_X \nu(dy) a(x, y) k(x, x) k(y, y) \\
&\leq C_5 \int_{\Lambda} \nu(dx) k(x, x) \int_{\Delta} \nu(dy) k(y, y) \\
&+ C_6 \int_{\Lambda} \nu(dx) k(x, x) \int_{\Delta^c} \nu(dy) a(x, y) < \infty.
\end{aligned} \tag{3.24}$$

Thus, by (3.20)–(3.24), the theorem is proven. \square

Theorem 3.2. (i) Let $s \in [\frac{1}{2}, 1]$, and let the conditions of Theorem 3.1 (i) be satisfied. Then the coefficients $d(x, \gamma)$ and $b(x, \gamma)$ defined by (2.12) and (2.13), satisfy condition (2.10). Thus, $\mathcal{FC}_b(C_0(X), \Gamma) \subset D(L_G)$, and for each $F \in \mathcal{FC}_b(C_0(X), \Gamma)$, $L_G F$ is given by formula (1.1).

(ii) Let $s \in [\frac{1}{2}, 1]$, and let the conditions of Theorem 3.1 (ii) be satisfied. Further assume that either

$$\forall \Lambda \in \mathcal{B}_0(X) \exists \Lambda' \in \mathcal{B}_0(X) \forall x \in \Lambda \forall y \in (\Lambda')^c : a(x, y) = 0, \quad (3.25)$$

or

$$\int_{\Delta} k(x, x)^2 \nu(dx) < \infty, \quad (3.26)$$

where Δ is as in Theorem 3.1 (ii). Then the coefficient $c(x, y, \gamma)$ defined by (2.14), satisfies condition (2.11). Thus, $\mathcal{FC}_b(C_0(X), \Gamma) \subset D(L_K)$, and for each $F \in \mathcal{FC}_b(C_0(X), \Gamma)$, $L_K F$ is given by formula (1.2).

Remark 3.3. If $X = \mathbb{R}^d$ and the function a is as in Remark 2.3, then condition (3.25) means that the function \tilde{a} has a compact support.

Proof of Theorem 3.2. We again prove only the part related to Kawasaki dynamics and only in the case $l = 1$, omitting the upper index (1) from our notation. We first assume that (3.25) is satisfied. Since the function a is bounded and satisfies (3.25), it suffices to show that, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\int_{\Lambda} \gamma(dx) \int_{\Lambda} \nu(dy) r(x, \gamma \setminus x)^{s-1} r(y, \gamma \setminus x)^s \chi_{\{r>0\}}(x, \gamma \setminus x) \chi_{\{r>0\}}(y, \gamma \setminus x) \in L^2(\mu). \quad (3.27)$$

We note that, for $s \in [\frac{1}{2}, 1]$, $2s - 1 \in [0, 1]$. Therefore, by the Cauchy inequality, we have

$$\begin{aligned} & \int_{\Gamma} \mu(d\gamma) \left(\int_{\Lambda} \gamma(dx) r(x, \gamma \setminus x)^{s-1} \chi_{\{r>0\}}(x, \gamma \setminus x) \right. \\ & \quad \times \left. \int_{\Lambda} \nu(dy) r(y, \gamma \setminus x)^s \chi_{\{r>0\}}(y, \gamma \setminus x) \right)^2 \\ & \leq \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \gamma(dx) r(x, \gamma \setminus x)^{2(s-1)} \chi_{\{r>0\}}(x, \gamma \setminus x) \\ & \quad \times \left(\int_{\Lambda} \nu(dy) r(y, \gamma \setminus x)^s \chi_{\{r>0\}}(y, \gamma \setminus x) \right)^2 \gamma(\Lambda) \\ & = \int_{\Gamma} \mu(d\gamma) \int_{\Lambda} \nu(dx) r(x, \gamma)^{2s-1} \chi_{\{r>0\}}(x, \gamma) \\ & \quad \times \left(\int_{\Lambda} \nu(dy) r(y, \gamma)^s \chi_{\{r>0\}}(y, \gamma) \right)^2 (\gamma(\Lambda) + 1) \end{aligned}$$

$$\begin{aligned}
&\leq \int_{\Gamma} \mu(d\gamma) \left(\int_{\Lambda} \nu(dx) (1 + r(x, \gamma)) \right)^3 (\gamma(\Lambda) + 1) \\
&\leq \left(\int_{\Gamma} \mu(d\gamma) \left(\int_{\Lambda} \nu(dx) (1 + r(x, \gamma)) \right)^6 \right)^{1/2} \left(\int_{\Gamma} \mu(d\gamma) (\gamma(\Lambda) + 1)^2 \right)^{1/2}. \tag{3.28}
\end{aligned}$$

By Lemma 3.1, we have, for each $n \in \mathbb{N}$,

$$\begin{aligned}
&\int_{\Gamma} \mu(d\gamma) \left(\int_{\Lambda} \nu(dx) r(x, \gamma) \right)^n \\
&= \int_{\Lambda} \nu(dx_1) \cdots \int_{\Lambda} \nu(dx_n) \int_{\Gamma} \mu(d\gamma) r(x_1, \gamma) \cdots r(x_n, \gamma) \\
&\leq \int_{\Lambda} \nu(dx_1) \cdots \int_{\Lambda} \nu(dx_n) \|r(x_1, \cdot)\|_{L^n(\mu)} \cdots \|r(x_n, \cdot)\|_{L^n(\mu)} \\
&\leq \frac{(2n)!}{2^n n!} \left(\int_{\Lambda} \nu(dx) k(x, x) \right)^n < \infty \tag{3.29}
\end{aligned}$$

Now, (3.27) follows from (3.28) and (3.29).

Next, we assume that (3.26) is satisfied. We fix $\Lambda \in \mathcal{B}_0(X)$ and denote

$$u(x, y) := a(x, y)(\chi_{\Lambda}(x) + \chi_{\Lambda}(y)).$$

Then, by the Cauchy inequality,

$$\begin{aligned}
&\int_{\Gamma} \mu(d\gamma) \left(\int_X \gamma(dx) \int_X \nu(dy) u(x, y) r(x, \gamma \setminus x)^{s-1} \chi_{\{r>0\}}(x, \gamma \setminus x) \right. \\
&\quad \left. \times r(y, \gamma \setminus x)^s \chi_{\{r>0\}}(y, \gamma \setminus x) \right)^2 \\
&\leq \int_{\Gamma} \mu(d\gamma) \int_X \gamma(dx) \int_X \nu(dy) u(x, y) r(x, \gamma \setminus x)^{2(s-1)} \chi_{\{r>0\}}(x, \gamma \setminus x) \\
&\quad \times r(y, \gamma \setminus x)^{2s} \chi_{\{r>0\}}(y, \gamma \setminus x) \int_X \gamma(dx') \int_X \nu(dy') u(x', y') \\
&= \int_{\Gamma} \mu(d\gamma) \int_X \nu(dx) \int_X \nu(dy) u(x, y) r(x, \gamma)^{2s-1} \chi_{\{r>0\}}(x, \gamma) \\
&\quad \times r(y, \gamma)^{2s} \chi_{\{r>0\}}(y, \gamma) \int_X (\gamma + \varepsilon_x)(dx') \int_X \nu(dy') u(x', y') \\
&\leq \int_{\Gamma} \mu(d\gamma) \int_X \nu(dx) \int_X \nu(dy) u(x, y) (1 + r(x, \gamma)) (1 + r(y, \gamma))^2 \\
&\quad \times \left(\int_X \gamma(dx') \int_X \nu(dy') u(x', y') + \int_X \nu(dy') u(x, y') \right).
\end{aligned}$$

By (2.15), it suffices to prove that

$$\int_{\Gamma} \mu(d\gamma) \left(\int_X \nu(dx) \int_X \nu(dy) u(x, y) (1 + r(x, \gamma)) (1 + r(y, \gamma)^2) \right)^2 < \infty, \tag{3.30}$$

$$\int_{\Gamma} \mu(d\gamma) \left(\int_X \gamma(dx) \int_X \nu(dy) u(x, y) \right)^2 < \infty. \quad (3.31)$$

We first to prove (3.31). We have, by Proposition 3.2,

$$\begin{aligned} & \int_{\Gamma} \left(\int_X \gamma(dx) \int_X \nu(dy) u(x, y) \right)^2 \\ &= \int_X \nu(dy) \int_X \nu(dy') \int_{\Gamma} \mu(d\gamma) \int_X \gamma(dx) \int_X \gamma(dx') u(x, y) u(x', y') \\ &= \int_X \nu(dy) \int_X \nu(dy') \int_{\Gamma} \mu(d\gamma) \left(\int_X \gamma(dx) u(x, y) u(x, y') \right. \\ &+ \left. \int_X \gamma(dx) \int_X (\gamma - \varepsilon_x)(dx') u(x, y) u(x', y') \right) \\ &= \int_X \nu(dy) \int_X \nu(dy') \left(\int_X \nu(dx) k(x, x) u(x, y) u(x, y') \right. \\ &+ \left. \int_X \nu(dx) \int_X \nu(dx') \left(\frac{1}{2} k(x, x')^2 + k(x, x) k(x', x') \right) u(x, y) u(x', y') \right) \\ &\leq \int_X \nu(dy) \int_X \nu(dy') \left(\int_X \nu(dx) k(x, x) u(x, y) u(x, y') \right. \\ &+ \left. \int_X \nu(dx) \int_X \nu(dx') \frac{3}{2} k(x, x) k(x', x') u(x, y) u(x', y') \right) \\ &= \int_X \nu(dy) \int_X \nu(dy') \int_X \nu(dx) k(x, x) u(x, y) u(x, y') \\ &+ \frac{3}{2} \left(\int_X \nu(dy) \int_X \nu(dx) k(x, x) u(x, y) \right)^2 \\ &\leq \int_{\Delta} \nu(dx) k(x, x) \left(\int_X \nu(dy) u(x, y) \right)^2 \\ &+ C_7 \int_X \nu(dy) \int_X \nu(dy') \int_X \nu(dx) u(x, y) u(x, y') \\ &+ \frac{3}{2} \left(\int_{\Delta} \nu(dx) k(x, x) \int_X \nu(dy) u(x, y) + C_7 \int_X \nu(dy) \int_X \nu(dx) u(x, y) \right)^2 < \infty. \end{aligned}$$

Next, we prove (3.30). By Lemma 3.1 and (3.26), we have

$$\begin{aligned} & \int_{\Gamma} \mu(d\gamma) \left(\int_X \nu(dx) \int_X \nu(dy) u(x, y) (1 + r(x, \gamma)) (1 + r(y, \gamma)^2) \right)^2 \\ &= \int_X \nu(dx) \int_X \nu(dx') \int_X \nu(dy) \int_X \nu(dy') u(x, y) u(x', y') \\ &\times \int_{\Gamma} \mu(d\gamma) (1 + r(x, \gamma)) (1 + r(x', \gamma)) (1 + r(y, \gamma)^2) (1 + r(y', \gamma)^2) \\ &\leq \int_X \nu(dx) \int_X \nu(dx') \int_X \nu(dy) \int_X \nu(dy') u(x, y) u(x', y') (1 + \|r(x, \cdot)\|_{L^4(\mu)}) \end{aligned}$$

$$\begin{aligned}
& \times (1 + \|r(x', \cdot)\|_{L^4(\mu)}) (1 + \|r(y, \cdot)\|_{L^4(\mu)}) (1 + \|r(y', \cdot)\|_{L^4(\mu)}) \\
& \leq C_8 \left(\int_X \nu(dx) \int_X \nu(dy) u(x, y) (1 + k(x, x)) (1 + k(y, y)^2) \right)^2 < \infty.
\end{aligned}$$

Thus, the theorem is proven. \square

4 Diffusion approximation

From now on, we set $X = \mathbb{R}^d$, $d \in \mathbb{N}$, and ν to be Lebesgue measure. We will show that, under an appropriate scaling, the Dirichlet form of the Kawasaki dynamics converges to a Dirichlet form which identifies a diffusion process on Γ having a permanent point process $\mu^{(l)}$ as a symmetrizing measure. The way we scale the Kawasaki dynamics will be similar to the ansatz of [15].

We denote by $\mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma)$ the space of all functions of the form (2.6) where $N \in \mathbb{N}$, $\varphi_1, \dots, \varphi_N \in C_0^\infty(\mathbb{R}^d)$ and $g \in C_b^\infty(\mathbb{R}^N)$. Here, $C_0^\infty(\mathbb{R}^d)$ denotes the space of smooth functions on \mathbb{R}^d with compact support, and $C_b^\infty(\mathbb{R}^N)$ denotes the space of all smooth bounded functions on \mathbb{R}^N whose all derivatives are bounded. Clearly,

$$\mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma) \subset \mathcal{F}C_b(C_0(\mathbb{R}^d), \Gamma),$$

and the set $\mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma)$ is a core for the Dirichlet form $(\mathcal{E}_K, D(\mathcal{E}_K))$.

We fix $s = 1/2$. Let us assume that the function $a(x, y)$ is as in Remark 2.3. Thus, the coefficient $c(x, y, \gamma)$ has the form

$$c(x, y, \gamma) = a(x - y) r(x, \gamma)^{-1/2} r(y, \gamma)^{1/2} \chi_{\{r>0\}}(x, \gamma) \chi_{\{r>0\}}(y, \gamma). \quad (4.1)$$

Note that $y - x$ describes the change of the position of a particle which hops from x to y . We now scale the function a as follows: for each $\varepsilon > 0$, we denote

$$a_\varepsilon(x) := \varepsilon^{-d-2} a(x/\varepsilon), \quad x \in \mathbb{R}^d. \quad (4.2)$$

The Dirichlet form $(\mathcal{E}_K, D(\mathcal{E}_K))$ which corresponds to the choice of function a as in (4.2) will be denoted by $(\mathcal{E}_\varepsilon, D(\mathcal{E}_\varepsilon))$.

Theorem 4.1. *Assume that the function a has compact support, and the value $a(x)$ only depends on $|x|$, i.e., $a(x) = \tilde{a}(|x|)$ for some function $\tilde{a} : [0, \infty) \rightarrow \mathbb{R}$. Further assume that the function $\varkappa(x, y)$ has the form $\varkappa(x - y)$ for some $\varkappa : \mathbb{R}^d \rightarrow \mathbb{C}$, and*

$$\lim_{y \rightarrow 0} \int_{\mathbb{R}^d} (\varkappa(x) - \varkappa(x + y))^2 dx = 0. \quad (4.3)$$

For each $l \in \mathbb{N}$, define a bilinear form $(\mathcal{E}_0, \mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma))$ by

$$\mathcal{E}_0(F, G) := c \int_\Gamma \mu^{(l)}(d\gamma) \int_{\mathbb{R}^d} dx r(x, \gamma) \langle \nabla_x F(\gamma \cup x), \nabla_x G(\gamma \cup x) \rangle. \quad (4.4)$$

Here

$$c := \frac{1}{2} \int_{\mathbb{R}^d} a(x) x_1^2 dx$$

(x_1 denoting the first coordinate of $x \in \mathbb{R}^d$), ∇_x denotes the gradient in the x variable, and $\langle \cdot, \cdot \rangle$ stands for the scalar product in \mathbb{R}^d . Then, for any $F, G \in \mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma)$,

$$\mathcal{E}_\varepsilon(F, G) \rightarrow \mathcal{E}_0(F, G) \quad \text{as } \varepsilon \rightarrow 0.$$

Remark 4.1. Assume that the function \varkappa is differentiable on \mathbb{R}^d . Denote

$$K(x, \delta) := \sup_{y \in B(x, \delta)} |\nabla \varkappa(y)|, \quad x \in \mathbb{R}^d, \quad \delta > 0.$$

Here $B(x, \delta)$ denotes the closed ball in \mathbb{R}^d centered at x and of radius δ . Assume that, for some $\delta > 0$,

$$K(\cdot, \delta) \in L^2(\mathbb{R}^d, dx). \quad (4.5)$$

Then condition (4.3) is clearly satisfied. Note that condition (4.5) is slightly stronger than the condition $|\nabla \varkappa| \in L^2(\mathbb{R}^d, dx)$.

Proof of Theorem 4.1. Again we will only present the proof in the case $l = 1$, omitting the upper index (1). We start with the following

Lemma 4.1. *Fix any $\Lambda \in \mathcal{B}_0(\mathbb{R}^d)$ and $\alpha \in (0, 1]$. Then, under the conditions of Theorem 4.1,*

$$r(x + \varepsilon y, \gamma)^\alpha \rightarrow r(x, \gamma)^\alpha \quad \text{in } L^2(\Gamma \times \Lambda \times \mathbb{R}^d, \mu(d\gamma) dx dy a(y)) \quad \text{as } \varepsilon \rightarrow 0.$$

Proof. We first prove the statement for $\alpha = 1$. Thus, equivalently we have to prove that

$$r(x + \varepsilon y, \gamma) \rightarrow r(x, \gamma) \quad \text{in } L^2(\Omega \times \Gamma \times \Lambda \times \mathbb{R}^d, \tilde{\mathbb{P}}(d\omega, d\gamma) dx dy a(y)) \quad \text{as } \varepsilon \rightarrow 0. \quad (4.6)$$

We have, using Jensen's inequality for conditional expectation,

$$\begin{aligned} & \int_{\Lambda} dx \int_{\mathbb{R}^d} dy a(y) \int_{\Omega \times \Gamma} \tilde{\mathbb{P}}(d\omega, d\gamma) (r(x + \varepsilon y) - r(x, \gamma))^2 \\ &= \int_{\Lambda} dx \int_{\mathbb{R}^d} dy a(y) \int_{\Omega \times \Gamma} \tilde{\mathbb{P}}(d\omega, d\gamma) \tilde{\mathbb{E}}(Y(x + \varepsilon y)^2 - Y(x)^2 \mid \mathcal{F})^2 \\ &\leq \int_{\Lambda} dx \int_{\mathbb{R}^d} dy a(y) \int_{\Omega \times \Gamma} \tilde{\mathbb{P}}(d\omega, d\gamma) (Y(x + \varepsilon y)^2 - Y(x)^2)^2 \\ &= \int_{\Lambda} dx \int_{\mathbb{R}^d} dy a(y) \int_{\Omega} d\mathbb{P} (Y(x + \varepsilon y)^4 + Y(x)^4 - 2Y(x + \varepsilon y)^2 Y(x)^2). \end{aligned} \quad (4.7)$$

Using the formula for moments of a Gaussian measure, we have

$$\begin{aligned}
& \int_{\Omega} Y(x + \varepsilon y)^4 d\mathbb{P} \\
&= 3 \left(\int_{\mathbb{R}^d} \varkappa(x + \varepsilon y - u)^2 du \right)^2 \\
&= 3 \left(\int_{\mathbb{R}^d} \varkappa(x - u)^2 du \right)^2 \\
&= \int_{\Omega} Y(x)^4 d\mathbb{P}.
\end{aligned} \tag{4.8}$$

Analogously, using condition (4.3) and the dominated convergence theorem, we get

$$\begin{aligned}
& \int_{\Lambda} dx \int_{\mathbb{R}^d} dy a(y) \int_{\Omega} d\mathbb{P} Y(x + \varepsilon y)^2 Y(x)^2 \\
&= \int_{\Lambda} dx \int_{\mathbb{R}^d} dy a(y) \left[\int_{\mathbb{R}^d} \varkappa(x + \varepsilon y - u)^2 du \cdot \int_{\mathbb{R}^d} \varkappa(x - u')^2 du' \right. \\
&\quad \left. + 2 \left(\int_{\mathbb{R}^d} \varkappa(x + \varepsilon y - u) \varkappa(x - u) du \right)^2 \right] \\
&\rightarrow \int_{\Lambda} dx \int_{\mathbb{R}^d} dy a(y) \int_{\Omega} d\mathbb{P} Y(x)^4 \quad \text{as } \varepsilon \rightarrow 0.
\end{aligned} \tag{4.9}$$

By (4.7)–(4.9), statement (4.6) follows.

To prove the result for $\alpha \in (0, 1)$, it is now sufficient to show the following

Claim. Let $(\mathbf{A}, \mathcal{A}, m)$ be a measure space and let $m(A) < \infty$. Let $f_{\varepsilon} \in L^2(m)$, $f_{\varepsilon} \geq 0$, $\varepsilon \in [-1, 1]$, and let $f_{\varepsilon} \rightarrow f_0$ in $L^2(m)$ as $\varepsilon \rightarrow 0$. Then, for each $\alpha \in (0, 1)$, $f_{\varepsilon}^{\alpha} \rightarrow f_0^{\alpha}$ in $L^2(m)$ as $\varepsilon \rightarrow 0$.

By e.g. [1, Theorems 21.2 and 21.4], $f_{\varepsilon} \rightarrow f_0$ in $L^2(m)$ implies that

- (i) $f_{\varepsilon} \rightarrow f_0$ in measure;
- (ii) $\sup_{\varepsilon \in [-1, 1]} \int f_{\varepsilon}^2 dm < \infty$;
- (iii) For each $\theta > 0$ there exist $h \in L^1(m)$ and $\delta > 0$ such that, for all $0 < |\varepsilon| \leq 1$ and for each $A \in \mathcal{A}$

$$\int_A h dm \leq \delta \Rightarrow \int_A f_{\varepsilon}^2 dm \leq \theta.$$

Hence, for $\alpha \in (0, 1)$, we get

- a) $f_{\varepsilon}^{\alpha} \rightarrow f_0^{\alpha}$ in measure;

$$\text{b) } \sup_{\varepsilon \in [-1,1]} \int f_\varepsilon^{2\alpha} dm \leq \sup_{\varepsilon \in [-1,1]} \int (1 + f_\varepsilon^2) dm < \infty;$$

c) Let θ , h , and δ be as in (iii). Set $h' := h + \frac{\delta}{\theta}$. Clearly, $h \in L^1(m)$. Assume that, for some $A \in \mathcal{A}$, $\int_A h' dm \leq \delta$. Hence $\int_A h dm \leq \delta$, and therefore $\int_A f_\varepsilon^2 dm \leq \delta$ for all $0 < |\varepsilon| \leq 1$. Furthermore, we get $\int_A \frac{\delta}{\theta} dm \leq \delta$, and therefore $m(A) \leq \theta$.
Now

$$\int_A f_\varepsilon^{2\alpha} dm \leq \int_A (1 + f_\varepsilon^2) dm \leq 2\theta.$$

Applying again [1, Theorems 21.2 and 21.4], we conclude the claim. \square

Fix any $F \in \mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma)$. We have

$$\begin{aligned} \mathcal{E}_\varepsilon(F, F) &= \frac{1}{2} \int_\Gamma \mu(d\gamma) \int_{\mathbb{R}^d} dx \int_{\mathbb{R}^d} dy \varepsilon^{-d-2} a((x-y)/\varepsilon) r(x, \gamma)^{1/2} r(y, \gamma)^{1/2} (F(\gamma \cup x) - F(\gamma \cup y))^2 \\ &= \frac{1}{2} \int_\Gamma \mu(d\gamma) \int_{\mathbb{R}^d} dx \int_{\mathbb{R}^d} dy a(y) r(x + \varepsilon y, \gamma)^{1/2} r(x, \gamma)^{1/2} \left(\frac{F(\gamma \cup \{x + \varepsilon y\}) - F(\gamma \cup x)}{\varepsilon} \right)^2. \end{aligned}$$

Assume that $0 < |\varepsilon| \leq 1$. Noting that the function F is local (i.e., there exists $\Delta \in \mathcal{B}_0(\mathbb{R}^d)$ such that $F(\gamma) = F(\gamma_\Delta)$ for all $\gamma \in \Gamma$) and that the function a has a compact support, we conclude that there exists $\Lambda \in \mathcal{B}_0(\mathbb{R}^d)$ such that

$$\begin{aligned} \mathcal{E}_\varepsilon(F, F) &= \int_\Gamma \mu(d\gamma) \int_\Lambda dx \int_{\mathbb{R}^d} dy a(y) r(x + \varepsilon y, \gamma)^{1/2} r(x, \gamma)^{1/2} \\ &\quad \times \left(\frac{F(\gamma \cup \{x + \varepsilon y\}) - F(\gamma \cup x)}{\varepsilon} \right)^2. \end{aligned} \quad (4.10)$$

By the dominated convergence theorem

$$r(x, \gamma)^{1/2} \left(\frac{F(\gamma \cup \{x + \varepsilon y\}) - F(\gamma \cup x)}{\varepsilon} \right)^2 \rightarrow r(x, \gamma)^{1/2} \langle \nabla_x F(\gamma \cup x), y \rangle^2 \quad (4.11)$$

in $L^2(\Gamma \times \Lambda \times \mathbb{R}^d, \mu(d\gamma) dx dy a(y))$ as $\varepsilon \rightarrow 0$. By Lemma 4.1 with $\alpha = 1/2$, (4.10) and (4.11)

$$\mathcal{E}_\varepsilon(F, F) \rightarrow \frac{1}{2} \int_\Gamma \mu(d\gamma) \int_\Lambda dx \int_{\mathbb{R}^d} dy a(y) r(x, \gamma) \langle \nabla_x F(\gamma \cup x), y \rangle^2. \quad (4.12)$$

Since $a(y) = \tilde{a}(|y|)$, for any $i, j \in \{1, \dots, d\}$, $i \neq j$, we have

$$\int_{\mathbb{R}^d} a(y) y_i y_j dy = 0$$

and

$$c = \frac{1}{2} \int_{\mathbb{R}^d} a(y) y_i^2 dy, \quad i = 1, \dots, d.$$

Therefore, by (4.12),

$$\mathcal{E}_\varepsilon(F, F) \rightarrow c \int_{\Gamma} \mu(d\gamma) \int_{\mathbb{R}^d} dx r(x, \gamma) |\nabla_x F(\gamma \cup x)|^2.$$

From here the theorem follows by the polarization identity. \square

We will now show that the limiting form $(\mathcal{E}_0, \mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma))$ is closable and its closure identifies a diffusion process.

In what follows, we will assume that the conditions of Theorem 4.1 are satisfied. We have

$$\begin{aligned} k(x, y) &= \int_{\mathbb{R}^d} \varkappa(x - u) \varkappa(y - u) du \\ &= \int_{\mathbb{R}^d} \varkappa(u - y) \varkappa(u - x) du = \int_{\mathbb{R}^d} \varkappa(u) \varkappa(u + y - x) du. \end{aligned}$$

Hence, by (4.3), the function $k(x, y)$ is continuous on $(\mathbb{R}^d)^2$. Thus, by Remark 3.1, $(Y(x))_{x \in X}$ is a Gaussian random field and formula (3.6) holds for all $(x, y) \in (\mathbb{R}^d)^2$.

Consider the semimetric

$$\begin{aligned} D(x, y) &:= \frac{1}{2} \left(\int_{\Omega} (Y(x) - Y(y))^2 d\mathbb{P} \right)^{1/2} \\ &= \frac{1}{2} (k(x, x) + k(y, y) - 2k(x, y))^{1/2} \\ &= \left(\int_{\mathbb{R}^d} \varkappa(u) (\varkappa(u) - \varkappa(u + y - x)) du \right)^{1/2}, \quad x, y \in \mathbb{R}^d. \end{aligned} \tag{4.13}$$

The associated metric entropy $H(D, \delta)$ is defined as $H(D, \delta) := \log N(D, \delta)$, where $N(D, \delta)$ is the minimal number of points in a δ -net in $B(0, 1) = \{x \in \mathbb{R}^d \mid |x| \leq 1\}$ with respect to the semimetric D , i.e., points x_i such that the open balls centered at x_i and of radius δ (with respect to D) cover $B(0, 1)$. The expression

$$J(D) := \int_0^1 \sqrt{H(D, \delta)} d\delta$$

is called the Dudley integral. The following result holds, see e.g. [4, Corollary 7.1.4] and the references therein.

Theorem 4.2. *Assume that $J(D) < \infty$. Then the Gaussian random field $(Y(x))_{x \in \mathbb{R}^d}$ has a continuous modification.*

Remark 4.2. Let \varkappa be as in Remark 4.1. Then, by (4.13), for any $x, y \in B(0, 1)$

$$\begin{aligned} D(x, y)^2 &\leq \|\varkappa(\cdot)\|_{L^2(\mathbb{R}^d, dx)} \left(\int_{\mathbb{R}^d} (\varkappa(u) - \varkappa(u + y - x))^2 du \right)^{1/2} \\ &\leq \|\varkappa(\cdot)\|_{L^2(\mathbb{R}^d, dx)} \|K(\cdot, 2)\|_{L^2(\mathbb{R}^d, dx)} |y - x|, \end{aligned}$$

where we assumed that $K(\cdot, 2) \in L^2(\mathbb{R}^d, dx)$. Then $J(D) < \infty$, see e.g. [4, Example 7.1.5].

Denote by $\ddot{\Gamma}$ the space of all multiple configurations in \mathbb{R}^d . Thus, $\ddot{\Gamma}$ is the set of all Radon $\mathbb{Z}_+ \cup \{+\infty\}$ -valued measures on \mathbb{R}^d . In particular, $\Gamma \subset \ddot{\Gamma}$. Analogously to the case of Γ , we define the vague topology on $\ddot{\Gamma}$ and the corresponding Borel σ -algebra $\mathcal{B}(\ddot{\Gamma})$.

Theorem 4.3. *Let $\varkappa(x, y)$ be of the form $\varkappa(x - y)$ for some $\varkappa \in L^2(\mathbb{R}^d, dx)$. Let $J(D) < \infty$. Let $l \in \mathbb{N}$ and $c > 0$. Then*

- (i) *The bilinear form $(\mathcal{E}_0, \mathcal{FC}_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma))$ defined by (4.4) is closable on $L^2(\Gamma, \mu^{(l)})$ and its closure will be denoted by $(\mathcal{E}_0, D(\mathcal{E}_0))$.*
- (ii) *There exists a conservative diffusion process*

$$M^0 = (\Omega^0, \mathcal{F}^0, (\mathcal{F}_t^0)_{t \geq 0}, (\Theta_t^0)_{t \geq 0}, (X^0(t))_{t \geq 0}, (P_\gamma^0)_{\gamma \in \ddot{\Gamma}})$$

on $\ddot{\Gamma}$ which is properly associated with $(\mathcal{E}_0, D(\mathcal{E}_0))$. In particular, M^0 is $\mu^{(l)}$ -symmetric and has $\mu^{(l)}$ as invariant measure. In the case $d \geq 2$, the set $\ddot{\Gamma} \setminus \Gamma$ is \mathcal{E}^0 -exceptional, so that $\ddot{\Gamma}$ may be replaced by Γ in the above statement.

Proof. We again discuss only the case $l = 1$, omitting the upper index (1). By (4.4), for any $F, G \in \mathcal{FC}_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma)$,

$$\begin{aligned} \mathcal{E}_0(F, G) &= c \int_{\Omega \times \Gamma} \tilde{\mathbb{P}}(d\omega, d\gamma) \int_{\mathbb{R}^d} dx \tilde{\mathbb{E}}(Y(x, \omega)^2 | \mathcal{F}) \langle \nabla_x F(\gamma \cup x), \nabla_x G(\gamma \cup x) \rangle \\ &= \int_{\Omega \times \Gamma} \tilde{\mathbb{P}}(d\omega, d\gamma) \int_{\mathbb{R}^d} dx Y(x, \omega)^2 \\ &\quad \times \langle \nabla_x (F(\gamma \cup x) - F(\gamma)), \nabla_x (G(\gamma \cup x) - G(\gamma)) \rangle. \end{aligned} \tag{4.14}$$

Fix $(\omega, \gamma) \in \Omega \times \Gamma$. Denote

$$f(x) := F(\gamma \cup x) - F(\gamma), \quad g(x) := G(\gamma \cup x) - G(\gamma).$$

Clearly, $f, g \in C_0^\infty(\mathbb{R}^d)$. In view of Theorem 4.2, $Y(x, \omega)^2$ is a continuous function of $x \in \mathbb{R}^d$. Hence, by [6, Theorem 6.2], the bilinear form

$$\mathcal{E}(f, g) := \int_{\mathbb{R}^d} \langle \nabla f(x), \nabla g(x) \rangle Y(x, \omega)^2 dx, \quad f, g \in C_0^\infty(\mathbb{R}^d),$$

is closable on $L^2(\mathbb{R}^d, |Y(x, \omega)|^2 dx)$. Now the closability of $(\mathcal{E}_0, \mathcal{F}C_b^\infty(C_0^\infty(\mathbb{R}^d), \Gamma))$ on $L^2(\Gamma, \mu^{(l)})$ follows by a straightforward generalization of the proof of [6, Theorem 6.3]. Part (ii) of the theorem can be shown completely analogously to [25, 29], see also [20]. \square

Remark 4.3. Heuristically, the generator of $(\mathcal{E}_0, D(\mathcal{E}_0))$ has the form

$$(LF)(\gamma) = \sum_{x \in \gamma} \left(\Delta_x F(\gamma) + \left\langle \frac{\nabla_x r(x, \gamma \setminus x)}{r(x, \gamma \setminus x)}, \nabla_x F(\gamma) \right\rangle \right).$$

Here, for $x \in \gamma$, $\nabla_x F(\gamma) := \nabla_y F(\gamma \setminus x \cup y)|_{y=x}$ and analogously Δ_x is defined. However, we should not expect that $r(x, \gamma)$ is differentiable in x .

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