AN INTERACTING PARTICLE PROCESS
RELATED TO YOUNG TABLEAUX

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ABSTRACT. We discuss a stochastic particle system consisting of a two-dimensional array of particles living in one space dimension. The stochastic evolution bears a certain similarity to Hammersley's process, and the particle interaction is governed by combinatorics of the Young tableaux.

CONTENTS

1. INTRODUCTION

The present note is related to our papers [4], [5], [6], and may be viewed as a companion to the latter paper.

The purpose of [4] and [6] was to construct new models of infinite-dimensional Markov dynamics of representation-theoretic origin.

In [4], we studied a four-parameter family of continuous time Feller Markov processes whose state space is the dual object $\widehat{U(\infty)}$ to the infinite-dimensional unitary group $U(\infty)$. The points of the space $\widehat{U(\infty)}$ parameterize the extremal characters of $U(\infty)$. Equivalently, the same space can also be viewed as the boundary $\partial\mathbb{GT}$ of the Gelfand–Tsetlin graph.

In [6], we described a similar construction related to the infinite symmetric group $S(\infty)$ that leads to a two-parameter family of Feller Markov processes on an infinite-dimensional cone $\widetilde{\Omega} \subset \mathbb{R}^\infty \times \mathbb{R}^\infty \times \mathbb{R}$, called the *Thoma cone*. The latter space is an amplified version of the dual object to $S(\infty)$: the base of the Thoma cone is the *Thoma simplex* $\Omega = \widehat{S(\infty)}$.

The paper [5] explains a striking resemblance between the character theory of the two groups, $U(\infty)$ and $S(\infty)$: we show how the characters of the latter group arise as a degeneration of those of the former group. In particular, we explain in what sense the Thoma cone can be viewed as a degeneration of $\partial\mathbb{GT}$. Moreover, the Markov dynamics on the Thoma cone can also be viewed as a degeneration of the Markov dynamics on $\partial\mathbb{GT}$.

The Markov processes from [4] and [6] may be viewed as interacting particle processes with *nonlocal* (or *long-range*) interaction. On the other hand, as shown in [4], each of the Markov processes on $\partial\mathbb{GT}$ is "covered" by a certain Markov process with *local* interaction, living on the path space of the graph $\mathbb{GT}$. In the present informal note we speculate that similar covering processes should exist in the case of the Markov dynamics on the Thoma cone, too.

Although we do not have a complete proof yet, we could not resist the temptation to introduce this model, because it provides an alternative approach to the results of [6].

The note contains no proofs and presents a collection of conjectures and claims based on some heuristic arguments.

2. Generalized standard Young tableaux

Denote $\mathbb{N} := \{1, 2, \dots\}$ and $\mathbb{N}^2 := \mathbb{N} \times \mathbb{N}$. We endow $\mathbb{N}^2$ with the structure of poset such that one element (i, j) of $\mathbb{N}^2$ is declared to be greater than another element (i', j') if $i \geq i'$, $j \geq j'$, and at least one of these inequalities is strict. By a (finite or infinite) Young diagram we mean a (finite or infinite) subset of $\mathbb{N}^2$ such that if it contains a given element of the poset $\mathbb{N}^2$ then it also contains all smaller elements.

Let $\mathbb{Y}$ denote the set of all finite Young diagrams including the empty one. We denote a Young diagram by the letter λ and write $|\lambda|$ for the size of λ. (Infinite Young diagram will appear later on.)

Recall that a *standard tableau* τ of a given shape $\lambda \in \mathbb{Y}$ is a bijection $\tau : \lambda \to \{1, 2, \dots, |\lambda|\}$ compatible with the partial order on λ inherited from $\mathbb{N}^2$. That is, $\tau(i, j) > \tau(i', j')$ whenever (i, j) is greater than (i', j').

Let $\mathbb{R}_{>0}$ stand for the set of strictly positive real numbers.

Definition 2.1. Following [5], we define a *generalized* standard Young tableau of a given shape $\lambda \in \mathbb{Y}$ as a function $h : \lambda \to \mathbb{R}_{>0}$ such that

- h is compatible with the partial order on λ, that is, $h(i, j) > h(i', j')$ whenever (i, j) is greater than (i', j'),
- the numbers $h(i, j)$ are pairwise distinct.

The conventional standard tableaux of a shape $\lambda \in \mathbb{Y}$ encode all possible ways of building λ step by step, starting with the empty set and adding an element (i, j) at each step. Likewise, a generalized standard diagram encodes a similar process, only we imagine that elements are added at some moments $h_1 < h_2 < \cdots < h_{|\lambda|}$ on the time axis $\mathbb{R}_{>0}$.

Definition 2.2. Given $r \in \mathbb{R}_{>0}$ and $\lambda \in \mathbb{Y}$, we denote by $H_r(\lambda)$ the set of all generalized standard tableaux h of the shape λ, such that $h(i, j) < r$ for all $(i, j) \in \lambda$. We agree that for the empty diagram $\varnothing$, the set $H_r(\varnothing)$ is a singleton. Next, we set

$$H_r := \bigsqcup_{\lambda \in \mathbb{Y}} H_r(\lambda).$$

Given $h \in H_r$, we write $\mathrm{sh}(h)$ for the corresponding Young diagram λ. It is convenient to define the "height" $h(i, j)$ outside λ, by setting $h(i, j) = r$ for all $(i, j) \in \mathbb{N}^2 \setminus \lambda$. Then we may interpret the elements $h \in H_r$ as the "height functions" $h : \mathbb{N}^2 \to (0, r]$ subject to the following conditions:

- $h(i, j) < h(i, j + 1)$ unless $h(i, j) = h(i, j + 1) = r$,

- likewise, $h(i, j) < h(i + 1, j)$ unless $h(i, j) = h(i + 1, j) = r$,

- the set $\mathrm{sh}(h) := \{(i, j) \in \mathbb{N}^2 : h(i, j) < r\}$ is finite,

- the numbers $h(i, j)$, where (i, j) ranges over $\mathrm{sh}(h)$, are pairwise distinct.

3. Markov dynamics on generalized tableaux

As above, we fix a number $r \in \mathbb{R}$. We are going to define a continuous time Markov process on H_r. It depends on two continuous parameters z and z' subject to the following condition:

Condition 3.1. Either both parameters z and z' are complex numbers with nonzero imaginary part and $z' = \bar{z}$, or both parameters are real and contained in an open unit interval of the form $(m, m + 1)$ for some $m \in \mathbb{Z}$.

This is equivalent to requiring that $(z + k)(z' + k) > 0$ for every $k \in \mathbb{Z}$. In particular, Condition 3.1 implies that

$$q(i, j) := (z + j - i)(z' + j - i) > 0, \qquad \forall (i, j) \in \mathbb{N}^2, \tag{3.1}$$

which is used in the sequel.

We need one more notation. Given $h \in H_r$ and $(i, j) \in \mathbb{N}^2$, we set

$$h^{\downarrow}(i, j) = \begin{cases} \max\{h(i-1, j), h(i, j-1)\}, & \text{if } i > 1 \text{ and } j > 1; \\ h(1, j-1), & \text{if } i = 1, \, j > 1; \\ h(i-1, 1), & \text{if } i > 1, \, j = 1; \\ 0, & \text{if } i = j = 1. \end{cases} \tag{3.2}$$

Let us fix a couple (z, z') of parameters satisfying Condition 3.1. We consider an infinite collection of independent Poisson processes indexed by the elements $(i, j) \in \mathbb{N}^2$, where each process is defined in the horizontal strip

$$D_r := \{(t, x) \in \mathbb{R}^2, \quad -\infty < t < +\infty, \quad 0 < x < r\}, \tag{3.3}$$

and the (i, j)th process has constant rate equal to $q(i, j)$ (see (3.1)). We denote by $\pi_r(i, j) \subset D_r$ the random point configuration from the (i, j)th process. We may assume that $\pi_r(i, j)$ has at most one point on each vertical line.

Definition 3.2. We introduce a Markov process $\widetilde{X}_r^{(z,z')}$ on H_r, whose time evolution is composed of two components, one deterministic and the other one stochastic:

- The deterministic component prescribes the height $h(i, j)$ (for every $(i, j) \in \mathrm{sh}(h)$) to grow with the varying velocity $v(y) := y(y + 1)$ depending on the current value $y := h(i, j)$, until the height attains the maximum value r, where it stabilizes.
- The stochastic component, on the contrary, forces the height to instantly drop to some level. More precisely, given $(i, j) \in \mathbb{N}^2$, the height $h(i, j)$ drops precisely at those time moments t for which the vertical line with abscissa t contains a point $(t, x) \in \pi_r(i, j) \subset D_r$ such that the ordinate x satisfies the constraints $h^{\downarrow}(i, j) < x < h(i, j)$. Then the height instantly takes the new value equal to x.

Note that the deterministic evolution preserves the set H_r: this follows from the very definition of H_r and the fact that the velocity $v(x)$ is a strictly increasing function in x. Likewise, the stochastic evolution also preserves H_r, by virtue of the requirement that for the drop to take place x must be greater than $h^{\downarrow}(i, j)$. More pedantically, we had to include in the above definition the additional requirement that a jump $h(i, j) \to x$ is forbidden whenever x is equal to $h(i', j')$ for some $(i', j') \in \mathrm{sh}(h)$ distinct from (i, j), but such an event has probability 0.

We may interpret $\widetilde{X}_r^{(z,z')}$ as an evolution of a system of particles on $(0, r]$ indexed by the two-dimensional array $\mathbb{N}^2$. It is worth noting that the stochastic component of the process includes an interaction between the particles, because the lower bound $h^{\downarrow}(i, j)$ for the possible drop of the (i, j)th particle depends on the positions of the neighboring particles. Here the "neighbors" are defined as the particles with neighboring indices $(i - 1, j)$ and $(i, j - 1)$. This kind of interaction is similar to that encountered in the exclusion processes.

On the other hand, because of the presence of two components, deterministic and stochastic, the process $\widetilde{X}_r^{(z,z')}$ may be viewed as a member of the class of *piecewise deterministic Markov processes* introduced by Davis [7].

The following claim looks very plausible to us.

Conjecture 3.3. The process $\widetilde{X}_r^{(z,z')}$ on H_r does not explode and so has infinite life time almost surely.

4. Evolution of Gibbs measures

Here we discuss a connection between the process $\widetilde{X}_r^{(z,z')}$ just defined and the process $X_r^{(z,z')}$ discussed in our papers [6] and [3].

Let us recall the definition of $X_r^{(z,z')}$ (see [6, Definition 8.9]). This is a continuous time jump Markov process with the state space $\mathbb{Y}$. The evolution is given the following Q-matrix (the matrix of jump rates):

$$Q_r^{(z,z')}(\lambda, \lambda + \square) = r(z + c(\square))(z' + c(\square))\frac{\dim(\lambda + \square)}{(|\lambda| + 1)\dim\lambda}, \quad \square \in \lambda^+,$$

$$Q_r^{(z,z')}(\lambda, \lambda - \square) = (r+1)\frac{|\lambda|\dim(\lambda - \square)}{\dim\lambda}, \quad \square \in \lambda^-, \tag{4.1}$$

$$-Q_r^{(z,z')}(\lambda, \lambda) = (2r+1)|\lambda| + rzz'.$$

The notation is the following: λ is a Young diagram; $\lambda \pm \square$ is another diagram obtained from λ by appending/removing a box $\square$; $\lambda^\pm$ is the set of those boxes that can be appended to (respectively, removed from) λ; $Q(\lambda, \lambda \pm \square)$ is the rate of the jump $\lambda \to \lambda \pm \square$; finally, $-Q(\lambda, \lambda)$ is equal to the sum of the rates of all possible jumps $\lambda \to \lambda \pm \square$.

As shown in [6], this Q-matrix is regular, meaning that the corresponding jump Markov process does not explode.

Let us return to the sets $H_r(\lambda)$ introduced in Definition 2.2. For every $\lambda \in \mathbb{Y}$, let $\mathrm{Tab}(\lambda)$ denote the finite set consisting of all (conventional) standard tableaux of the shape λ. As seen from Definition 3.2, there is a natural projection $H_r(\lambda) \to \mathrm{Tab}(\lambda)$. For every standard tableau $\tau \in \mathrm{Tab}(\lambda)$, the fiber of this projection over τ can be viewed as an open simplex $\Delta(\tau)$ of dimension $N = |\lambda|$, formed by the ordered N-tuples of reals $(x_1, \ldots, x_N)$ such that $0 < x_1 < \cdots < x_N < r$. It follows that $H_r(\lambda)$ can be viewed as a bounded open subset of $\mathbb{R}^N$ whose closure $\overline{H_r(\lambda)}$ is a convex polytope endowed with a triangulation.

For instance, if $\lambda = (2, 1)$, then $N = 3$, $H_r(\lambda)$ consists of the triples $(x_1, x_2, x_3) = (h(1,1), h(1,2), h(2,1))$ subject to the conditions

$$0 < x_1 < r, \quad 0 < x_2 < r, \quad 0 < x_3 < r, \quad x_1 < x_2, \quad x_1 < x_3, \quad x_2 \neq x_3,$$

and $\overline{H_r(\lambda)}$ is obtained by removing the last inequality and making the remaining inequalities weak. The polytope $\overline{H_r(\lambda)}$ is the union of two closed simplices, which are singled out by the inequalities $x_2 \leq x_3$ and $x_2 \geq x_3$, respectively.

Definition 4.1. Recall (see Definition 2.2) that H_r is the disjoint union of the sets $H_r(\lambda)$, where λ ranges over $\mathbb{Y}$. Following [6] we define a *Gibbs measure* on H_r as a probability measure such that its restriction to each subset $H_r(\lambda)$ is proportional to the Lebesgue measure. The set of all Gibbs measures is denoted by $\mathscr{G}_r$.

Obviously, the natural projection $H_r \to \mathbb{Y}$ establishes a one-to-one correspondence between $\mathscr{G}_r$ and the set $\mathscr{M}(\mathbb{Y})$ of all probability measures on $\mathbb{Y}$.

Claim 4.2. *The process $\widetilde{X}_r^{(z,z')}$ preserves the set $\mathscr{G}_r$ of Gibbs measures, and the evolution of the Gibbs measures induced by the process $\widetilde{X}_r^{(z,z')}$ coincides, under the*

bijection $\mathscr{G}_r \to \mathscr{M}(\mathbb{Y})$, with the evolution of the probability measures on $\mathbb{Y}$ induced by the process $X_r^{(z,z')}$.

Actually, we can rigorously prove only the infinitesimal version of the claim: the application of the infinitesimal generator of $\widetilde{X}_r^{(z,z')}$ to a Gibbs measure translates to the application of the generator of $X_r^{(z,z')}$ to the corresponding measure on $\mathbb{Y}$.

Remark 4.3. Here is a simple yet curious formal identity used in the proof. Assume we are given a standard tableau of the shape $\mathbb{N}^2$, that is, a total order on the set $\mathbb{N}^2$ compatible with its partial order. Next, for every $(i,j) \in \mathbb{N}^2$, set

$$(i,j)^{\downarrow} := \begin{cases} \max\left\{(i-1,j),(i,j-1)\right\}, & \text{if } i > 1 \text{ and } j > 1, \\ (1,j-1), & \text{if } i = 1, j > 1, \\ (i-1,1), & \text{if } i > 1, j = 1, \\ \text{undefined}, & \text{if } (i,j) = (1,1), \end{cases}$$

and

$$(i,j)^{\uparrow} := \min\left\{(i+1,j),(i,j+1)\right\},$$

where the maximum and minimum are taken relative to the prescribed total order on $\mathbb{N}^2$. Finally, attach to every $(i,j) \in \mathbb{N}^2$ a formal variable $y(i,j)$. Then the identity in question is

$$\sum_{(i,j) \in \mathbb{N}^2} \left(y((i,j)^{\uparrow}) + y((i,j)^{\downarrow}) - 2y(i,j)\right)(z+j-i)(z'+j-i) = 2\sum_{(i,j) \in \mathbb{N}^2} y(i,j)$$

with the agreement that $y((1,1)^{\downarrow}) := 0$.

As explained in [6], the process $X_r^{(z,z')}$ has a stationary distribution, the so called *mixed z-measure*, denoted by $M_r^{(z,z')}$. Let $\widetilde{M}_r^{(z,z')}$ denote the corresponding Gibbs measure on H_r. Claim 4.2 implies that $\widetilde{M}_r^{(z,z')}$ serves as the stationary distribution for the process $\widetilde{X}_r^{(z,z')}$.

Claim 4.2 says that the process $\widetilde{X}_r^{(z,z')}$ in some sense "covers" the jump process $X_r^{(z,z')}$. Note that Young diagrams $\lambda \in \mathbb{Y}$ can be represented as particle configurations, so that $X_r^{(z,z')}$, like $\widetilde{X}_r^{(z,z')}$, can also be interpreted as an interacting particle process. However, a substantial difference between $\widetilde{X}_r^{(z,z')}$ and $X_r^{(z,z')}$ is that the particle interaction is local in the former process and highly non-local in the latter one. In this sense, $\widetilde{X}_r^{(z,z')}$ seems to be simpler than $X_r^{(z,z')}$.

We showed in [6] that for every pair $r' > r$ there exists a "link" $\Lambda_r^{r'}$ (an infinite stochastic matrix of format $\mathbb{Y} \times \mathbb{Y}$), which intertwines the processes $X_{r'}^{(z,z')}$ and $X_r^{(z,z')}$ for all (z,z'). For the covering processes the picture is simpler in the sense that the processes $\widetilde{X}_{r'}^{(z,z')}$ and $X_r^{(z,z')}$ are "linked" by an ordinary map, the truncation map (5.1) that we now define.

5. Dynamics on infinite generalized tableaux

Given $r' > r$, we define the *truncation map* $H_{r'} \to H_r$ as the transform

$$h(i,j) \to \min(h(i,j), r), \qquad \forall (i,j) \in \mathbb{N}^2. \tag{5.1}$$

Obviously, for a triple $r'' > r' > r$, the composed map $H_{r''} \to H_{r'} \to H_r$ is the same as $H_{r''} \to H_r$. Therefore, we may define the projection limit space

$$H := \varprojlim H_r, \qquad r \to +\infty.$$

The elements of H can be viewed as the functions $h : \mathbb{N}^2 \to \mathbb{R}_{>0} \cup \{+\infty\}$ such that

- $h(i,j) < h(i+1, j)$ unless $h(i,j) = h(i+1,j) = +\infty$;
- $h(i,j) < h(i, j+1)$ unless $h(i,j) = h(i,j+1) = +\infty$;
- the finite values $h(i,j)$ are pairwise distinct and do not have accumulation points on $\mathbb{R}_{>0}$.

Note that the natural projection $H \to H_r$ is still given by (5.1). Note also that, for $h \in H$, the set

$$\mathrm{sh}(h) := \{(i,j) \in \mathbb{N}^2 : h(i,j) < +\infty\} \subseteq \mathbb{N}^2$$

is a (possibly infinite) Young diagram.

Every element $h \in H$ may be interpreted as a system of particles on the extended halfline $\mathbb{R}_{>0} \cup \{+\infty\}$, indexed by $\mathbb{N}^2$ (we assume that the (i,j)th particle has coordinate $x = h(i,j)$). If $\mathrm{sh}(h) = \mathbb{N}^2$, then all particles are on $\mathbb{R}_{>0}$, but it may happen that $\mathrm{sh}(h)$ is a proper subset of $\mathbb{N}^2$; then there is an infinite reservoir of particles at infinity, indexed by the elements of $\mathbb{N}^2 \setminus \mathrm{sh}(h)$. An important requirement included in the definition is that the particle configuration on $\mathbb{R}_{>0}$ is always locally finite meaning that there are finitely many particles in any bounded interval.

Claim 5.1. *The family of Markov processes $\{\widetilde{X}_r^{(z,z')} : r \in \mathbb{R}_{>0}\}$ is consistent with the truncation maps $H_{r'} \to H_r$ and so there exists a unique Markov process $\widetilde{X}^{(z,z')}$ on the space H, consistent with this family by means of the truncation maps $H \to H_r$.*

In other words, $\widetilde{X}^{(z,z')}$ can be viewed as the projective limit of the processes $\widetilde{X}_r^{(z,z')}$.

The Markov dynamics of such a system, given by the process $\widetilde{X}^{(z,z')}$, is described in the same way as for the truncated processes $\widetilde{X}_r^{(z,z')}$, see Section 3 above, only the open interval $(0, r)$ should be replaced by the halfline $\mathbb{R}_{>0}$, and the endpoint r is shifted to $+\infty$.

Informally, in the particle system interpretation, the deterministic component looks as the accelerating movement of particles to the right with velocity $v(y) = y(y+1)$, while the the concurrent stochastic component forces the particles to instantly jump to the left. The latter component is driven by a two-dimensional array $\{\pi(i,j) : (i,j) \in \mathbb{N}^2\}$ of independent Poisson processes in the half-plane $D := \{(t, x) : t \in \mathbb{R}, x > 0\}$, where the (i,j)th process has constant rate $q(i,j)$ given by (3.1). Note that the particles can escape from $\mathbb{R}_{>0}$ to infinity and return back to $\mathbb{R}_{>0}$.

Note also that the sample trajectories of $\widetilde{X}^{(z,z')}$ look more sophisticated than those of the truncated process $\widetilde{X}_r^{(z,z')}$. This is due to a major difference between the Poisson processes in the strip D_r and in the half-plane D: the pushforward of a Poisson configuration under the projection to the t-axis is locally finite in the former case but not in the latter case.

Now we extend our definition of Gibbs measures to the space H.

Definition 5.2. A probability measure on H is said to be a *Gibbs measure* if its pushforward under the truncation map $H \to H_r$ is a Gibbs measure on H_r in the sense of Definition 4.1, for every $r \in \mathbb{R}_{>0}$. The set of all Gibbs measures on H is denoted by $\mathscr{G}$.

The above discussion shows that the process $\widetilde{X}^\infty$ preserves the Gibbs measures.

On the other hand, as explained in [5], there is a one-to-one correspondence $\mathscr{G} \leftrightarrow \mathscr{M}(\widetilde{\Omega})$, where $\widetilde{\Omega}$ is an infinite-dimensional cone in $\mathbb{R}^\infty \times \mathbb{R}^\infty \times \mathbb{R}$, called the *Thoma cone*, and $\mathscr{M}(\widetilde{\Omega})$ denotes the space of probability measures on $\widetilde{\Omega}$. Further, we showed in [6] that the family $\{X_r^{(z,z')} : r \in \mathbb{R}_{>0}\}$ of Markov processes on $\mathbb{Y}$ determines, via the links $\Lambda_r^{r'}$, a Markov process $X^{(z,z')}$ on the Thoma cone $\widetilde{\Omega}$. Because of the bijection $\mathscr{G} \leftrightarrow \mathscr{M}(\widetilde{\Omega})$, this leads to the following conclusion:

Claim 5.3. *The process $\widetilde{X}^{(z,z')}$ covers the process $X^{(z,z')}$ in the sense that the evolution of arbitrary probability measures on the Thoma cone induced by $X^{(z,z')}$ is the same as the evolution of the Gibbs measures on H induced by $\widetilde{X}^{(z,z')}$.*

Here is a reformulation. Given a point $\omega \in \widetilde{\Omega}$, let us denote by G_ω the Gibbs measure on H corresponding to the Dirac measure at ω. The correspondence $\omega \mapsto G_\omega$ can be viewed as a Markov kernel $\Lambda_H^{\widetilde{\Omega}} = \Lambda_H^{\widetilde{\Omega}}(\omega, dh)$. Then the above claim means that $\Lambda_H^{\widetilde{\Omega}}$ intertwines $X^{(z,z')}$ and $\widetilde{X}^{(z,z')}$.

In [6], we showed that the process $X^{(z,z')}$ on the Thoma cone has a stationary distribution $M^{(z,z')}$, which is defined by the family $\{M_r^{(z,z')} : r \in \mathbb{R}_{>0}\}$ via the links $\Lambda_r^{r'}$. Let $G^{(z,z')}$ stand for the corresponding Gibbs measure on H; in other words, $G^{(z,z')}$ is the transform of $M^{(z,z')}$ by $\Lambda_H^{\widetilde{\Omega}}$.

Claim 5.4. *The Gibbs measure $G^{(z,z')}$ serves as a stationary distribution for $\widetilde{X}^{(z,z')}$.*

6. Concluding remarks

Besides the truncation maps $H \to H_r$ there exist other projections under which the Markov property of the process $\widetilde{X}^{(z,z')}$ is not destroyed. Namely, one may fix an arbitrary Young diagram, finite or infinite, and focus on those particles that are indexed by the elements (i, j) of that diagram.

The simplest such example is obtained when the diagram in question consists of the single element $(1, 1)$. Then the model represents a single particle that moves on

the extended halfline $\mathbb{R}_{>0} \cup \{+\infty\}$. The dynamics is described exactly as before: a deterministic movement to the right with the velocity $v(y) = y(y+1)$ combined with instant jumps to the left directed by the Poisson process in the half-plane D with constant rate $q(1,1) = zz'$.

A more complicated model arises when we take the infinite one-row diagram $\{(1, j) : j \in \mathbb{N}\}$. Then we have countably many particles, enumerated by the numbers $j \in \mathbb{N}$ and occupying positions $x_1 < x_2 < \ldots$ on $\mathbb{R}_{>0}$ with a possible infinite reservoir at infinity, meaning that it may happen that $x_j = +\infty$ for all j large enough. The dynamics in this model has a certain resemblance with the *Hammersley process* studied in Aldous-Diaconis [1].

Finally, note that the process $\widetilde{X}^{(z,z')}$ admits a *Plancherel-type* degeneration in the spirit of [6, Section 10]. The idea is to scale the space variable x by substituting $x(zz')^{-1}$ instead of x, and pass to a limit as both z and z' go to $+\infty$. Then we get a simplified model, where the velocity in the deterministic component depends on the coordinate $y \in \mathbb{R}_{>0}$ via $v(y) = y$, and the backward jump rules are the same for all particles (they are governed by independent Poisson processes in D with constant rate $q = 1$).

7. Acknowledgements

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